Build Operate Transfer: A Concrete Study of Public Private Partnerships

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ABSTRACT

LIU, YIYI: Build Operate Transfer: A Concrete Study of Public Private Partnerships.
(Under the direction of Gary Biglaiser)

This dissertation focuses on one of the most used Public-Private Partnership (PPP) contracts: Build-Operate-Transfer (BOT) contract to study the theoretical problems haven't been well addressed in the contract design. Chapter one focuses on the applicability of a general recognized method in PPP contract design. The conventional goal of contract design is to find the best possible arrangement between a principal and an agent from the principal's point of view. While this goal readily permits the principal to modify her options of an instrument, it treats her decision path as a black box. For principals who make decisions based on complementary collaborations among departments, proper institutional arrangements to assist better decision-making are of special importance. Departmental collaborations may create challenges for the proper application of otherwise sound approaches to find the optimal contract, such as, the Revelation Principle, which requires the agents' truthful and comprehensive reporting of their private information. While presuming that the principal has control over the entire mediation plan, this study finds that, in certain situations, cooperative principals with restricted power over the entire mediation plan should be cautious about making decisions relying only on the relevant information and the optimality of the coordinated results. It also shows that the full report of the agents' private information is key to a valid application of the revelation principle, regardless of the report's decision-relevance. Even with a full report, an entire mediation plan made by coordinated principals with restricted control may not achieve the best outcome, because they treat each other's decisions, while complementary, as sunk. Chapter One ends with regulatory suggestions concerning institutional design in institutions with multi-level cooperation.

Chapter 2 uses BOT contract to study how the government efficiently resorts to the private sector firm's assistance for public goods' provision. BOT contract stipulation is a two-dimensional screening process for both the constructional and operational regulatory decisions. The optimal BOT contract extracts the private contractor's relative operational efficiency to compensate his upfront construction cost investment. Different from the literature, if the contractor only has one-dimensional high efficiency, "no distortion at top" for neither complementary decision is guaranteed. The prior distribution over types guides the decision of rationing out the least efficient type to reduce rent paying. A newly proposed Lagrangian decomposition method in combinatorial optimization studies is used to justify the optimal screening process. The government's utility is non-separable over the construction and operation decisions, hence the sovereign decision process potentially incurs a tradeoff between decisions independence and decision efficiency.

Chapter 3 studies the liability allocation between partners when a negative shock hits in a Public-Private Partnership. In a PPP contract, a private sector firm trades its upfront investment and cost for a time-constrained de facto monopoly of the public good; while the government trades off the regulation intensity on the monopoly power for the gain of the public service provision. Given the presence of dual tradeoffs, the liability concern in a Public-Private Partnership and that in traditional partnerships differ. First, the government makes no initial investment but has the final ownership of the project. Such ownership empowers the government the rights to claim the residual value of the public project as a creditor. Second, when a negative shock hits, the private sector's financial ability and its advantage on construction technology of the project provision prevents the private contractor from taking the full responsibility. This paper focuses on how to design PPP contracts that incentivize a high quality provision of the public good, concerning the randomness of the project's revenue stream and the liability allocations when an unfavorable event happens. Unlike in a traditional partnership, the loss of the private sector in a PPP is not up to the limit of the investment they put in the public project but up to the limit of the gain they get from the public project's revenue.

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CHAPTER 1

RETHINKING THE REVELATION PRINCIPLE IN THE INSTITUTINAL DESIGN FOR PPPS

1.1 Introduction

Fifty countries signed the article of agreement for the Asian Infrastructure Investment Bank (AIIB) on June 29, 2015. Signifying both the economic and political importance of multilateral cooperation on public service provision around the world, AIIB was initiated through governmental cooperation and modeled on the practice of existing organizations like the World Bank, IMF and the Asian Development Bank. Their organizational practices and introduction of private sector participation in the provision of public services deserve particular attention. In order to serve its mission of bringing such partnership to the next level of healthy sustainability, AIIB will have to be innovative regarding the design mechanisms of multilateral cooperation. This paper addresses these two issues from a theoretical point of view and provides regulatory suggestions.

Before diving into a purely theoretical discussion, it is useful to describe the current practice of public and private sector cooperation. A developed contract system called Public-Private Partnerships (PPP) has been guiding this practice. Build-Operate-Transfer (BOT), the most utilized PPP contract, captures the features of the private provision of public services. In a BOT contract, the selected entity constructs and operates the public service during the granted phase, then transfers the project back to the government afterward due to the project's public nature. BOT in essence is a financial agreement between the government and a private sector entity. Relieving governmental budget limits on construction and utilizing private sector operational advantages during the granted phase are obvious benefits of BOT contracts. Moreover, from a new PPP market perspective, the private sector firm benefits through initial construction costs and technology investment by becoming a de facto monopoly power while operating the public services during the granted duration of a BOT contract. In the meantime, the government is given time to find the best contract to balance the need for provision of public goods and the control of the selected private sector firm's monopoly power.

This paper simulates the stipulation of BOT contracts. Two independent government agencies make construction and operation decisions separately to select a private contractor with potentially various cost structures. With the new understanding of BOT contracts, this paper revisits two important assumptions of the canonical revelation principle in mechanism design: player's need to report private information completely and the mechanism designer's need for full control over the entire mediation plan. Through the assumptions implicit to the government's role in stipulating BOT contracts, this paper offers insights regarding informational requirements and organizational arrangements necessary for providing the optimal public-private partnerships through BOT contract.

By introducing two different reporting schemes of a private contractor's cost-structure, I study the effects of the so-called Chinese Wall on the optimal BOT contract design. The Chinese Wall is a natural restriction on the information disclosure in the two-department institutional setup [1]. Such restriction aims to clear foggy jurisdictional boundaries between different agencies and prevent each department's independent discretionary power from administrative interferences. Without uncertainty, the discretionary power of the departments does not obscure the optimality of the independent decision of each department in the BOT contract design. Ideally, Chinese Walls prevent big consortiums from monopolizing the entry into the contract and provide opportunities for small businesses with certain advantages to gain the right to either construction or operation in the BOT contract

[1] The ethical barrier between different divisions of a financial (or other) institution to avoid conflict of interest. For example, a Chinese Wall is said to exist between the corporate-advisory area and the brokerage department of a financial services firm to separate those giving corporate advice on takeovers from those advising clients about buying shares.

[2]. As a matter of fact, uncertainty potentially constrains the optimality of the departments' discretionary power. The two-department institutional setup demands the optimal BOT contract is a combinatorial optimal outcome. Incomplete information disclosure to each department under the Chinese Wall coarsens the each department's criterion of selecting the contractor.

Even disregarding the effect of the information disclosure on a BOT contract design, a two-department institutional setup essentially creates a new common agency problem with preference-aligned principals (Principals make independent complementary decisions and share the same objective function). There is a potential direct efficiency loss due to the two-department institutional setup. In the literature, common agency problems are often due to rivalry concerns among principals.[3] In such environments, each principal partially affects the agent's utility and thus the agent has an extended message space with strategic concerns. Both factors lead to the failure of indexing the principal's strategy with the agent's type, therefore the failure of the revelation principle application. It is easy to suspect a causal relationship between the strategic concerns among rivalry principals and such failure. However, the new common agency problem in this model indicates that there is a possibility of failing to apply the Revelation Principle even with preference-aligned principals. Here, the tradeoff is choosing between guaranteeing the mechanism implementation efficiency via a high-power authority and focusing on a fairness concern through different principals' collaboration. This kind of tradeoff is very common in large organizations and governments.

In Section 2, I present the models with two independent regulatory agencies in a government that accepts different private information report schemes and compare the government's utility under those report schemes. In Section 3 and 4 I address why the

[2]Office of Management and Budget: Competition if Contracting - Contract Bundling... In particular, there is ongoing concern that agencies are unnecessarily bundling contracts and, in doing so, have created an environment that makes it difficult for small business to flourish.

[3] The Revelation and Delegation Principles in Common Agent Games, David Martimort and Lars Stole, 2002, Econometrica

environment involving Chinese Wall fails to properly apply the revelation principle from two different perspectives. Finally, I conclude in Section 5 with regulatory suggestions.

1.2 Literature Review

This paper studies a direct and indirect effect of the governments' institutional setup on the optimal concession contract design. The institutional setup of the government affects how the information is dislosed in a government concession. Institutional design in this paper specifically means a setup where two government regulatory agencies collaboratively make complementary decisions for the contract in consideration without communication. This assumption restrains attentions to possible sequential communication equilibria.

I first present how the literature distinguishes the sequential equilibrium and sequential communication equilibrium. I then review the literature that associates mechanism design with the coarse theorem to emphasize the important function of incentives related property rights in the design of Build-Operate-Transfer contracts. I further show how the literature perceives the applicability of the Revelation Principle and explain the how the institutional setup in this paper affects the application of the Revelation Principle. Different from the extant literature, the concrete institutional setup in this paper breaks a presumption of the Revelation Principle. The fundamental reason of the application failure lies in the individual agency's constrained control over the entire mechanism. This paper also perceives such failure as a result of a common agency problem. The broad discussion in the literature of the common agency problem concerns only rivalry principals, and how such feature results in the failure of the revelation principle. This paper shows even with cooperative regulatory agencies in the government as co-mechanism designer, the institutional setup prevents a viable implementation environment for the revelation principle.

Kreps and Wilson (1982) points out that a sequential equilibrium is without communication and Myerson's sequential communication equilibrium is equilibrium for multistage games with communication. Myerson (1985) distinguishes the information disclosure for the mechanism designer and that to the player. For the player, broader information disclosure induces more disobedient behaviors against the mechanism designer's directions. The information of "name of the action or move" revealing to the player is sufficient for the designer to implement the optimal plan in a communication mechanism. In a multi-stage sequential communication game, Myerson acknowledges the difficulty to prevent the players taking advantage of the new information revealed at each stage and the difficulty of well defining a rational behavior for the players when Bayesian updating is impossible. Even though this paper does not allow new information coming to the player at different stages. But different information disclosure processes arising from the institutional setup in the two-sector game would create similar difficulties to find sequential communication equilibrium.

Literature sees that wasteful allocations come form the incentive constraints in mechanism design. By using the Revelation Principle, a direct mechanism maximizes expected revenue with information disclosure, and there is a positive probability of wasteful allocation due to the information asymmetry. With the concern of how the institutional setup obscures the information disclosure (besides the concern *when*the information asymmetry resolves), there could also be wasteful allocations. The full ex-post efficiency of resource cannot be guaranteed attainable not only because of the uncertainty of the information but also the inefficiency due to the way information is disclosed.

There is a fundamental relationship between the theory of mechanism design and the domain of applicability of Coarse Theorem. The above concerns of the information disclosure (when and how) can both be interpreted as the "transaction costs" that may potentially invalidate the Coarse Theorem, hence prohibit a profitable bilateral trade. Cramton, Gibbons and Klemperer (1987) consider that incentive-compatible mechanism can be designed with symmetric assignment of property right to grant the object to the individual who has higher values for it. This paper interprets their idea differently in the public- private partnership environment. In such partnerships, the government values the provision of the public goods most and the private sector values the gain of the monopoly power of the operational right of the public goods most. They trade off their benefits to the realization of the other's at the same time. Hence how to realize a profitable trade is as important as when profitable trade is possible.

Lewis and Sappington (1988) consider a more general concept: countervailing incentives to see that "redistribution of property rights tends to reduce the welfare losses caused by incentive constraints". This is an important concern in the public private partnerships. Because the ownership of public goods usually belongs to the government, hence if redistributions can improve welfare, therefore the privatization of public goods like the previously promoted deserves more appreciation. However, this is not the case as I show in the model. The ownership of the public goods is an important instrument for the government to introduce the private provision of the public goods.

It is very intuitive to use the Revelation Principle to find the optimal mechanism, hence the Revelation Principle has a wide recognition. Myerson (1988) conducts surveys on the application of the Revelation Principle. The Revelation Principle says a direct-revelation mechanism is any mechanism that has a complete type representation for the strategy set. He realizes that there is no equivalent first-price sealed-bid auction for incentive-compatible direct-relation mechanism in all situations because of the change of the exogenous environments. Wilson (1985) also appreciates that the Revelation Principle helps to design mechanisms of many different Bayesian collective-choice problems with good properties. Yet the Revelation Principal has implicit assumptions on the selection of equilibriums and also the communication structure of a mechanism. In this paper, I interpret the communication structure as a process of information disclosure.

Myerson (1988) talks out the mechanism selection problem. When we consider bargaining games in which individuals can bargain over mechanisms, there should be no loss of generality in restricting our attention to equilibria in which there is one incentive-compatible mechanism that is selected with probability one independently of anyone's type –inscrutability principle. However, the inscrutability principle does not imply that the possibility of revealing information during a mechanism-selection process is irrelevant.

We should not expect the individuals in a mechanism-selection game to inscrutably agree to an incentive-efficient mechanism that implicitly puts as much weight.

The failure of the Revelation Principle in this paper's setting can also be perceived as the result of a common agency problem. When we study games where two principals i and j contract with an agent and principal i cannot contract over the set of allocations controlled by principal j. Our central interest is the richness of the underlying message space. When the Revelation Principal is applied, principals prune off-equilibrium strategies, and indexing the remaining strategy set with agent's types. The literature realizes that the Revelation Principle fails to apply in common agent problems with rivalry concern the principals' side. There is no one to one relationship between the strategy space and the agent's type space because the competition among principals complicates the strategy space. This paper extends the common agency literature by showing a missing one-to-one relationship between cooperative principals. In this paper, the principals have aligned preferences, but restricted power over the entire mediation plan. This paper shows that with preference-aligned principals of limited influences in a communication game, none of the principals wants to index their strategies only with the agent's decision relevant private information because doing so will constrain themselves in a subset of the entire communication game.

McAfee (1993) proves that the Revelation Principle can be applied in a space where each principal can enlarge the agent's type space to take care of the rivalry concerns between principals and such a sequence of enlargements converges to a universal type space.

Epstein and Peter (1999) show that the Revelation Principle is valid in games with a universal message space. Such a message space incorporates the market information (rivalry concerns between principals) and the agent's types into a sufficiently rich language.

Martimort and Stole (2002) introduces the delegation principle that implements the truthful equilibrium through decentralization. The idea is to restrict principal's strategy to coarsened choices among the original equivalence classes and the agent is allowed to choose the implementation probability distribution over the subsets of offered equivalent classes. Hence the mechanism design shifts focus from the communication per se to the equilibrium allocations in such game. The constraint on the message space of the communication game can always be translated into relevant probability measures over actions.

1.3 The Model

In this section, I present a two-stage communication games differ only in the reporting schemes. Every BOT contract is determined by the construction and operation agencies' complementary decisions. According to the Chinese Wall's informational restriction, the contractor just reports decision-relevant cost to the agencies accordingly. I compare that to the reporting scheme of full report of the cost structure to both agencies. The results of two different reporting schemes are presented in parallel for each agency's decision. Proposition 1 summarizes two reporting schemes' effects on the operation agency's decisions. Proposition 2 and Proposition 3 summarize the construction agency's different decisions under those reporting schemes. The purpose of the parallel comparison is first to see how the informational restriction of the Chinese Wall constrains the agencies' choice sets. Section 3 goes into details on this point. In short, it is not appropriate to use the revelation principle in an environment with the information restriction on the reports because an untruthful equilibrium can arise and generate better outcomes compared to the best truthful equilibrium. Then we also want to see how the organizational structure implied by the Chinese Wall– two independent agencies making complementary decisions- limits the incentive each agency can give to the private contractor. Section 4 compares the optimal mechanism with two independent agencies to that of one. The results illustrate the inefficiency due to the Chinese Wall's structural implication.

Setup

The government conducts the construction and operation concession for a public project. There is one contractor with private information about its construction (F_i) and operation cost (c^j). The government's construction and operation regulators make independent decisions without communication to each other, but they share the same objective function.

When the same agent is contracted to build and operate the project which is returned to the government at some later date, we say the contract is a Build-Operate-Transfer (BOT) contract.

The instruments of the construction regulator are monetary transfer T_i and construction probability P_i. The instruments for the operation regulator are monetary transfer O^j and authorized phase t^j.

The project of lifespan n has a stable revenue flow during the operation stage: R if operated by the contractor and s if operated by the government.

The private information held by the contractor is the contractor's cost structure, including its operation and construction cost (c^j, F_i) and the prior of such type is α_{ji}; the contractor is either high (H) or low (L) efficiency type on both cost dimensions.

I assume there are two different reporting requirements by the regulators: 1) payoff-relevant information only and 2) a complete report of all the private information. When the contractor only reports decision relevant cost to different agencies, we say the information restriction of the Chinese Wall is placed.

The timing of the two-stage game is the construction regulator announces the construction contract (T_i, P_i) and the contractor reports its payoff-relevant construction cost F_i (or full report (c^j, F_i)); the operation regulator announces the operation contract $(t^j, O^j, Q^j)(Q^j$ is the probability the contractor (c^j, F_i) is given the project with the contract $(t^j, O^j))$ and the contractor reports its payoff-relevant operation cost c^j (or full report (c^j, F_i)).

In the next section, I show that the revelation principle fails to hold in the first reporting scheme because it compresses the information conveyed to the regulator therefore prevents the truthful implementation of the best available outcome. I prove such failure by constructing an untruthful equilibrium which cannot be supported by the optimal truthful contract, hence restricting to the direct mechanism is not sufficient for the regulators to pursue the best outcome achievable.

The Government's Problem Under Two Different Reporting Schemes

In this section, by looking at two regulatory agencies' problems separately, I compare the government's utility under two different reporting schemes. I show that the construction regulator can do better with full private information reported even though the operation cost is irrelevant for the construction regulator and the construction regulator cannot affect the operation stage's decision.

Operation Regulator

Using backward induction, I start with the operation regulator's problem. The solution to such problem is a function of the optimal construction regulator's contract (T_i, P_i) and the operator's beliefs about which type enters into the operation stage.

In the case of full reporting, these beliefs are:

$$\alpha^H = \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}} \quad \text{and} \quad \alpha^L = \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

In the case of partial reporting (decision-relevant information only), the operation regulator's beliefs are:

$$\alpha^H = \frac{\sum_i \alpha_{Hi}}{\sum_i \sum_j \alpha_{ji}} \quad \text{and} \quad \alpha^L = \frac{\sum_i \alpha_{Li}}{\sum_i \sum_j \alpha_{ji}}$$

Recall, Q^j is a randomization probability of choosing different operational stage contracts, the operation sector's program is:

$$
\begin{aligned}
\max_{(\mathbf{t},\mathbf{O},\mathbf{Q})} \sum_{j\in\{H,L\}} & \alpha^j[Q^j(n-t^j)s+(1-Q^j)n\cdot s-O^j] \\
& s.t. \\
\forall j,k \in \{H,L\} & \\
Q^j[t^j(R-c^j)+O^j] \geq & Q^k[t^k(R-c^j)+O^k] \qquad \text{(IC)} \\
Q^j[t^j(R-c^j)+O^j] \geq & 0 \qquad \text{(IR)} \\
0 \leq & t^j \leq n \\
0 \leq & Q^j \leq 1
\end{aligned}
$$

Proposition 1. *Given the beliefs α^H and α^L, the operation regulator chooses to let both high and low operation cost types operate for sure when the contractor is much more operationally efficient than the government, that is: $R - s > c^L + \frac{\alpha^H n(c^L - c^H)}{\alpha^L}$; otherwise the low operational efficient contractor and the high one are excluded from the operation contract successively as the relative efficiency $(R - s)$ shrinks.*

The solutions to the operation regulator's problem take the same pattern in both reporting schemes: at the operation stage, the operation regulator optimally extracts the contractor's relative operational efficiency. However, they are not identical since beliefs differ across the reporting schemes.

I bring the optimal solution to the operation regulator's program back to the construction regulator's problem, then solve for the optimal solution for the entire two-stage game.

The condition is more complicated in analysis when the relative efficiency between the government and the private contractor shrinks. This paper only uses the best case scenario where the government can utilize both level of operational efficiency types to focus on explaining the efficiency lost from the Chinese Wall's information restriction.

Construction Regulator

The construction regulator's program:

$$\max_{(\mathbf{P},\mathbf{T})} \sum_{i\in\{H,L\}} \alpha_{ji}[-T_i + Q^j(n-t^j)s + (1-Q^j)n\cdot s - O^j]P_i$$

$$s.t.$$

$$\forall i, m \in \{H, L\}$$

$$[t^{j*}(R-c^j) + O^{j*} + T_i - F_i]P_i \geq [t^{k*}(R-c^j) + O^{k*} - T_m - F_i]P_m \tag{IC}$$

$$[t^j(R-c^j) + O^j + T_i - F_i] \geq 0 \tag{IR}$$

$$0 \leq P_i \leq 1$$

Under the full reporting scheme, we have:

Proposition 2. *When the prior probability of (c^L, F_L) is low and the information rent on the operation dimension is high:*

$$\alpha_{LL} < \frac{F_L - F_H}{n(R-c^L) - F_H} \quad \textit{and} \quad n(c^L - c^H) - (F_L - F_H) > 0,$$

the optimal construction stage contract satisfies:

- *The type (c^L, F_L) is excluded from the contract, $P(c^L, F_L) = 0$.*
- *The monetary transfers paid to different types are: $T(c^j, F_i) = F_H, j = H, L; i = H$.*

Under the restricted information revelation scheme, where only payoff-relevant (for the construction regulator) is reported, we have:

Proposition 3. *The construction regulation agency's optimal decision depends on the expected value of the project $\alpha_L(n\,s - F_L)$ after the rent concerns $\alpha_H(F_L - F_H)$, where $\alpha_i = \sum_j \alpha_{ji}$ with $i, j = H, L$:*

- *When the value of the project is large enough, $\alpha_L(n\,s - F_L) - \alpha_H(F_L - F_H) > 0$: all types are asked to construct, $P(c^j, F_i) = 1$; every type receives the same compensation: $T(c^j, F_i) = F_L$.*

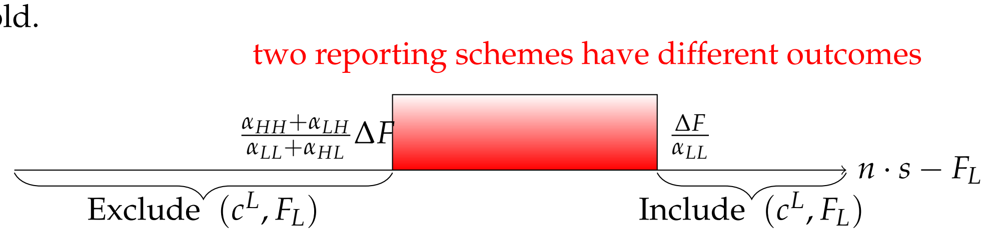
- *When the value of the project is low, $\alpha_L(n\,s - F_L) - \alpha_H(F_L - F_H) < 0$, the low type(s) on the construction dimension are excluded from the contract, $P(c^j, F_L) = 0$, high constructional efficient type receive zero rent $T(c^j, F_H) = F_H, j = H, L$.*

Propositions 2 and 3 describe what the construction regulator does when the contractor is sufficiently more operationally efficient than the government (i.e. the inequality on Proposition 1 holds: $R - s > c^L + \frac{\alpha^H n(c^L - c^H)}{\alpha^L}$).

Propositions 2 and 3 also tell us that based on its believes, the operation regulator let whoever constructed the project operates it, for type ji, $Q^j = 1$ if and only if $P_i > 0$.

When the contractor only needs to report its construction cost F_i to the construction regulator, type (c^L, F_L) constructs if $(\alpha_{LL} + \alpha_{HL})(n \cdot s - F_L) > (\alpha_{LH} + \alpha_{HH})(F_L - F_H)$, which says the benefit of including the low constructional efficient type is greater than the cost of paying information rent. Yet, when the contractor reports a full summary of its private information, type (c^L, F_L) constructs if $n(R - c^L) - F_L > (1 - \alpha_{LL})(n(R - c^L) - F_H)$, since the construction regulator is forward looking, it considers rent-paying on both dimensions.

In the full-report scheme, two things affect the construction regulator's decision. First, the construction regulator knows whomever constructs will operate. Second, the construction regulator has identical preferences with the operation regulator. Knowing the operation regulator plays a best response, the best that the construction regulator can do is to ration out the type (c^L, F_L) when $n(R - c^L) - F_L > (1 - \alpha_{LL})(n(R - c^L) - F_H)$ does not hold.



Define $n \cdot s - F_L$ (the government operates on its own and the private contractor constructs) as the minimum of the project value the government expects to realize by hiring a contractor (c^j, F_i). In the shaded region, with the full report of the private sector's cost structure, the construction regulator is able to do better by rationing the type (c^L, F_L) out to save construction information rent. In particular if α_{LL} is "small" and α_{LH} is "large" (the contractor is more likely to be a type (c^H, F_L) and unlikely to be (c^L, F_L)), it is not optimal for the construction regulator to pool the contracts for types (c^L, F_L) and (c^L, F_H). If the contractor only need to report partial cost structure to different agencies accordingly, types (c^L, F_L) and (c^L, F_H) can not be distinguished. Though the Chinese Wall is intended to promote the fairness concern, offering opportunity for small business with certain advantages to participate in public service provision. Restricting both the information released to different agencies and independent agencies' decision seem reasonable under the Chinese Wall. Yet the results of this section show that instead of encouraging small business with certain advantages to participate in the Public-Private Partnership contracts like BOT model, the Chinese Wall's negative effect on contract stipulation is more significant because it attracts inefficient private contractors to the partnership. The following two sections further explain how the Chinese Wall affects the stipulation of the optimal BOT contract and seek reasonable solutions to resolve inefficiency caused by the informational and structural restriction of the Chinese wall among the regulation agencies while preserve its fairness concerns of the partnership.

1.4 Restricted Information Disclosure: An Indirect Effect

With a Chinese Wall, it seems sufficient that both government agencies receive a restricted report to make their independent decisions for a BOT contract. The construction regulator only requires the construction cost and the operation regulator only needs the operation cost. Yet in this two-sector model, a BOT contract that relies on the agent's partial truthful report would hurt the government. Both regulatory agencies using the revelation principle would separate types in a more coarse way than with a full report. Chinese Walls are intended for protection of small private business with certain advantages, yet its information restriction on the reporting scheme would prevent the government from selecting the best BOT. This section shows that with the Chinese Wall's information requirement, untruthful equilibrium can rise and generate better outcomes than the optimal truthful outcome in the reporting game. The way to achieve the optimal outcome while preserving the protection for small private business is to use menu game instead.

In a communication game with competing principals, any single principal fails to index their strategies only with the types of the agent. Because there are strategic concerns among principals and with the agent, the message space is enlarged by the market information, which are composed of other principals' contracts. The principal is not able to use the revelation principle properly, thus they may work on designing payoff equivalent menu game. We may expect that if principals coordinate, the revelation principle can be applied without the strategic concerns. Yet the two-sector BOT stipulation game with restricted report suggests otherwise. For coordinated government agencies (they have the same objective function) with limited reports on the private sector's cost structure, indexing their strategies only with the decision relevant private information would constrain the agencies in a subset of the entire communication game. Therefore, aiming to excavate partial truth is weakly strategically dominated. Since the wall is with the ethical concern to protect the small business with certain advantages, a reasonable way to address this issue is to focus on the payoff-equivalent menus the government can offer with such constrained information.

I show here that a menu offered by regulation agencies yields an equivalent payoff for the government as in the full-report case. In a reporting game, this equilibrium can be reached only if the private contractor reports untruthfully. The menu is: if the contractor ever constructs, it will get the compensation as F_H; if the contractor ever operates, it will pary for a franchise fee $n \cdot (R - c^L)$ and let it operate for the entire length of the project n. The necessary parameter conditions for the menu are: 1) the contractor's operation efficiency is high enough: $R - s > c^L + \frac{\alpha^H n(c^L - c^H)}{\alpha^L}$; 2) the contractor is unlikely to be (c^L, F_L): $\alpha_{LL} < \frac{F_L - F_H}{n(R - c^L) - F_H}$; and 3) the information rent on the operation dimension is higher than that on the construction dimension $n(c^L - c^H) - (F_L - F_H) > 0$.

Under these conditions, an untruthful equilibrium exists in the reporting game: the government agencies pool the contracts as if all types were (c^L, F_H); type (c^H, F_L) always has the incentive to lie about its construction cost, and get the information rent difference of the construction and operation dimensions $n(c^L - c^H) - (F_L - F_H)$.The other types (are indifferent to) tell the truth. Any message conveyed between the regulation agencies and the private contractor that result in this untruthful equilibrium is within the same payoff equivalent class of the menu game. In the appendix, I show both government agencies are best responding to the agent's reports and different types are best responding to the announced contracts. This untruthful equilibrium generates a better outcome for the government because the low efficiency type on both dimensions (c^L, F_L) is excluded out of the contract. The government needs not to pay the information rent on the construction dimension. The optimal contract for both regulatory agencies with partial report cannot achieve this outcome.

The completeness of the reported information is key to the regulators' best plan if the regulators are interested in reporting games. Reporting games like commonly used auctions in government's concessions require at least the winner's truthful report. With a constrained report of private information satisfying the Chinese Wall's information restriction, the truth doesn't guarantee the government an optimal outcome by using revelation principle. The partial truth the regulator gathered might seem sufficient for the present planning, while the unrevealed part of the private information would indeed affect the player's future utility and therefore make the current plan inadequate. Under such circumstance, the government regulators should focus on the payoff equivalent menus to achieve the best outcome achievable and therefore eliminate the negative effect of the restricted message conveyed.

1.5 Constrained Power of Two Regulators: A Direct Effect

In the process of stipulating a BOT contract, different agencies' complementary partial control over the agent's utility is another force that fails a proper application of the revelation principle. With the Chinese Wall's information restriction, sometimes there is no consistent belief supporting the truthful equilibrium that generates the same utility for the government as its better untruthful counterpart. Even without the information restriction (namely the private contractor report the full cost structure to both agencies), the split of decision process would impair the agencies' ability to offer incentives to the private contractor. If there is only one regulator stipulating the entire BOT contract, the regulator can always punish the agent hardly later on to resolve the inefficiency at the early stage or vice versa.

For independent regulatory agencies with constrained power over the entire partnership agreement, there exists another implicit tradeoff between the application of the revelation principle and the strategic form of the multi-stage game.[4] For example, in the two-stage game of stipulating a BOT contract, there is efficiency loss because the operation agency cannot affect the construction agency's decision. Hence, any construction cost is sunk to the operation agency. I show this by comparing the optimal contract of two regulatory agencies with full report to that of a single regulator in a certain parameter space. The revelation principle fails in this case because no beliefs can support the truthful equilibrium of the two regulators' collaborative optimal contract to generate the same utility for the government as a single regulator can do.

The certain parameter condition is summarized as: 1) the contractor's operation efficiency is high enough: $R - s > c^L + \frac{\alpha^H n(c^L - c^H)}{\alpha^L}$; 2) the contractor is less likely to be (c^L, F_L): $\alpha_{LL} < \frac{n(c^L - c^H)}{n(R - c^H) - F_L}$; and 3) the information rent on the operation dimension is lower than that on the construction dimension $n(c^L - c^H) - (F_L - F_H) < 0$. With the single regulator's optimal contract, every type is treated as if they were (c^H, F_L) type and in that case the government is able to not pay the operational information rent. In order to generate the payoff equivalent outcome, if the contractor constructs, the construction agency compensates the contractor $F_L - n(c^L - c^H)$; the operation regulator let the contractor operate

[4] Myerson also mentions "conceptual tradeoff between the revelation principle and the generality of the strategic form" of the multi-stage game. Game Theory: Analysis of Conflict

the entire length n and charge zero franchise fee if the contractor operate. Recall, the operation regulation agency's objective function is $\alpha^j[Q^j(n-t^j)s+(1-Q^j)n\cdot s-O^j]$. In the two-stage game, the construction cost is sunk cost to the operation regulator. No consistent belief of the operation regulator (even with full cost-structure report) is able to support "charge zero franchise fee and let the contractor operate the entire length of the operation stage" as the operation stage component of the two agencies' collaborative optimal contract.

The strategic form of the two-stage model makes construction cost payoff-irrelevant for the operation regulator. Such game structure assumption implicitly makes the construction cost sunk and explicitly reduces the government's utility in our two-stage model. The assumption's implication on the partnership between government agencies and private contractors is more profound. Ideally government agencies can rigorously implement contracts based on a complete report of the private contractor's unrevealed cost structure. But it is seldom the case that any independent agency has full control over a policy plan. Different agencies are supposed to work as one social planner through transparent communication to improve the social benefits. Yet each agent's limited control over the entire policy results in the potential inability to provide the private sector proper incentives to induce the best outcome. The uncontrollable part of decision-making can make any agent's decision a latent dominated one. Then it is very useful to distinguish to what extent the government's organization itself (how different government agencies work together) would cause such kind of institutional costs. In the above BOT stipulation game, such cost is incurred by the independent agencies assumption. If construction and operation agencies work together as one social planner, the social planner can achieve better outcomes when the construction information rent is higher. Nevertheless, it is also important to remember that the two independent decision processes (Chinese Walls) are to protect small business with certain advantages, and the information transparency between agencies is not sufficient for the optimal BOT design. Coherent cooperation is also needed. By clearly distinguishing the Chinese Wall's effects of its informational restrictions from that of its structural constraints, the government is able to choose different mechanism modes to offer BOT contract while preserving both the efficiency and the fairness.

Using Section 3 and Section 4, I show that the two-sector institutional setup has a direct effect on the optimal BOT contract desgin. The constrained power of individual regulators would potential make one department treat the other department's decision as sunk. The two-sector institutional setup also has indirec effect on optimal BOT contract desgin. The two-sector institutional setup may justify some restricted infomation disclosure (e.g. Chinese Wall), hence restrict the regulator's choice and coarsen the regulator's selection criterion of the contractor.

Moreover, as we interpret the two-sector insitutional setup as two regulatory agencies are selecting a common contractor agency. I show that regulators' full control over the entire mediation plan and the comprehensive awareness of the private sectors' private information are required to implement the optimal truthful mechanism, even with coordinated principals in the common agency problem. In the next section, I conclude with the policy suggestions.

1.6 Conclusion

BOT contracts have been widely used to finance budget-constrained governments at the construction stage and the government can cross-subsidize the private contractor with the revenue generated at the operation stage. Applications of BOTs on infrastructural projects have demonstrated that the private provision for public services is viable. Yet implementing the project through BOT doesn't guarantee that the contract is preferable due to the potential over-compensation of private sector firms. Setting considerations of corruption aside, the selection of an inefficient private sector firm may arise from departmental collaboration in stipulating contracts. Hence it is important to set up proper institutions to further assist the optimal contracting and the prudent selection of private sector firms.

As I showed in the previous sections, the two-sector institutional setup has both direct and indirect effects on optimal BOT contract design. Since the indirect effect comes from the way the institutional setup affects the private information disclosure in the contract stipulation. Contractor's partial information disclosed to the regulators strictly weakens the optimality of the contract design. Even with complete information disclosure, the two-sector institutional setup still has an indirect effect on the optimal BOT contract design. The constrained control of individual regulators changes their perspectives of the contract. Any contract decision that they do not have control with, the individual regulator tends to treat that as sunk. Consistent believes are needed to justify the change of game structure. There is a tradeoff between implementing the contract with high efficiency through one high-power regulator versus a fairness concern via a two-sector institutional setup. Neither institutional setup dominates.

We understand the direct and indirect effects through how the conventional Revelation Principle can be properly applied to find the optimal BOT contract.

In theory, preference-aligned regulators only capable of extracting partial truth should not use the Revelation Principle. The regulators' complementary decisions of extracting partial truth are (weakly) strategically dominated by those of full truth extraction despite the truth's decision relevance. The Revelation Principle fails to apply properly here because there is no type representative of every strategy on the equilibrium path. Yet even with comprehensive truth, preference-aligned regulators with constrained power may not be able to implement the best contract for they may lack the ability to provide sufficient incentives. Under this condition, the institutional design faces a tradeoff between the effectiveness of a central regulator and the practical collaborations between regulatory agencies. The Revelation Principle fails because the relationship of type representation and the regulators' strategy profile is broken by the game structure that is directly affected by the institutional setup of two sectors.

In practice, unavoidable multilateral collaboration among different regulatory agencies and the intended fairness concern behind the informational and organizational restrictions (like Chinese Wall) are seemingly natural institutional products in large organizations like governments, multilateral development banks, etc. While optimal contract designing demands an ideal institutional setup, those products' inability to fulfill the institutional requirements will potentially impair each regulatory agency's ability to offer sufficient incentives to the private sector firms hence actually inhibiting the optimal contracting in a public-private partnership. A somewhat more delicate point is that the unfulfilled requirements are not always optimality-relevant. Therefore the institutional design innovation for BOT contracting comes from distinguishing different situations.

Often times, we recognize the importance of dynamic contracting between the government and large organizations, but we may forget the institutional setup should not be stationary as well. Adopting flexible decision modes in different situations is neither arbitrary, nor does it create rent-seeking possibilities, but it is significant for setting up the institutions for optimal contract designing.

The institutional design here faces a tradeoff between the information dictatorship of the central power and the fairness concerns of the departments' collaboration. In some situations, high-powered authority is necessary for stipulating more effective partnership plans. In other situations, the collaboration among independent regulatory agencies is more desired due to the fairness concern or a pure practical demand.

In order to avoid inefficiencies suggested by multilateral cooperation, several things need special attention. First of all, the government should always require a full report of the contractor's complete cost structure; reporting decision-relevant cost alone leaves space for inefficiency. Second, focusing on the payoff equivalent menu is a better solution than relying on the agents' truthful reports. Third, different agencies within a large organization should require a transparent communication system to exchange information and such transparency should be implemented through coherent cooperation and consistent supervision. Otherwise agencies can communicate with irrelevant information, deliberately missing important portions in order to possess the resource to seek rent and such behavior may not have legal consequences. Lastly and most importantly, regulators should be cautious about the special type of sunk cost faced by large organizations.

CHAPTER 2

APPLYING COMBINATORIAL OPTIMIZATION IN MULTI-DIMENSIONAL SCREENING

2.1 Introduction

Nowadays, the governments, the traditional suppliers of public services, face budget constraints and technology constraints in innovations of such services. Therefore returning to private assistance becomes a new solution. PPP (Public-Private-Partnership) contracts evolve under such trend. BOT (Build-Operate-Transfer) is one of the most used PPP contract form and captures all the essence of a public-private partnership. In a Build-Operate-Transfer (BOT) contract, the government decides whether the selected contractor can construct the project and decides how long such contractor can operate the project and generate revenue to compensate the upfront cost and investment. The government's optimal BOT contract is to provide a screening menu for the contractor to self-identify its cost-structure. In general, the contractor's participation constraint is IR, and its self-identification constraints are ICs. The government's screening problem is to maximize its objective function subject to constraints IC, IR by choosing its regulatory instruments. Hence we can perceive this as a monopoly regulation problem with two-dimensional private information.

The problem of missing a natural ordering on multi-dimensional space inhibits a general method for multi-dimensional screening. When we have a special case of BOT contract design, where the dimensionalities of the private information is the same of the number of the allocation instruments the regulator could use, we may attempt to simplify the multi-dimensional screening problem by decomposing the problem into sub-problems with potentially easier solutions.

In a BOT contracting environment, the type of the contractor (its cost structure: including construction and operation costs) is privately binary distributed on two dimensions. There are two allocation instruments the government can use (construction and operation decisions). Naturally we might suspect that if we have two regulators cooperatively stipulate the BOT contract, we can translate the two-dimensional screening problem into two sub-screening problem on each dimension and apply the conventional approach to look at the relaxed problem and deal with the problem in a easier approach. Yet we can prove that such decomposition is invalid to translate the multi-dimensional screening problem into multiple one-dimensional screening problems. In one word, such decomposition coarsens the selection criterion for the contractor.

Using the combinatorial optimization method proposed by Kim (1986), the correct decomposition methods is to produce copies of the two-dimensional screening problem and analyze them on two subsets. The validity of such decomposition relies both on the preference of players as well as the game structure. We can prove that the second decomposition dominates the first one. We study conditions in which the single regulator's optimal mediation plan can be implemented by two independent regulators and how such menu is implemented. There is equivalence between the menu implementability and the decomposability of the single regulator's Lagrangian problem. I also study how the combinatorial game structure interacts with the uncertainty of the private contractor's cost structure, and their effects on the optimality of the contract design.

By using the combinatorial method proposed by Baunmann (2014), we understand the effect of the game structure on the optimal contract design, which offer a different perspective of solving multi-dimensional screening problem. Given the optimality of the problem, the decomposition problem is optimizing in the intersection of the convex hulls of the two regulators' constraint sets. In such convex hull, the operation regulator extracts the relative operational efficiency in every parameter space. And the construction regulator makes the construction decision based on the value of the project while taking into account of the extracted operational efficiency and the information rent on both dimensions. Such decomposition method is valid only when the two departments can observe each other decisions. If not, the single regulator's Lagrangian problem cannot be properly decomposed into two independent regulators' problems. Because the communication assumption alters the rule of the game, which would affect the equilibrium concepts used, eventually it affects the implementability of the proposed optimal contracts.

Uncertainty given by the prior distribution of the contractor's types determines both the optimal allocation and the actual contract form. Even though different regulatory agencies share the same objective, there is potential inefficiency implied by the institutional setup of two independent regulatory agencies.

This paper is arranged as following: I review previous literature on multi-dimensional screening in Section Two. Section Three presents two models with different reporting-schemes in the two-sector model. Each reporting scheme stands for a specific Lagrangian decomposition method. I then compare the optimal solutions of two different decomposition methods. Section Three presents a model of the single regulator and its optimal solutions. By comparing the optimal solution of the single regulator and that of the two-sector's cooperation, in Section Four I propose conditions under which a valid decomposition of the original Lagrangian problem can arise to simplify the solution to the original problem. Comparison between models and different legislation implications will be offered at the end of each section.

2.2 Literature Review

This paper studies a multi-dimensional screening problem under a concrete economic setup. I use Build-Operate-Transfer contract as a starting point to analyze the government's screening process in a public-private partnership. Two regulatory decisions: construction probability and operational length define a Build-Operate-Transfer contract. The firm has two dimensions of private information about its cost structure. The screening has to concern the non-separable utility over those two regulatory decisions as well as the tradeoff and efficiency concerns of those decisions.

Multi-dimensional screening has difficulties firstly arising from the ordering complexity over multi-dimensions. Wilson (1993), Armstrong (1996) and Rochet and Chone (1998) all talk about this technical difficulty in a continuous environment without a concrete closed-form solution to particular problems. Various methods have been proposed to take care of this issue and to offer more concrete solutions. Armstrong (1996) shows that in order to extract higher revenue from the high type, excluding low valuation consumers is usually optimal. Armstrong (1999) shows that in a price-discrimination environment (an environment to best extract consumer surplus), higher dimensions of taste distribution helps to design mechanisms. Armstrong and Vickers (1998) and Rochet and Stole (1998) show that competition intensity may also help to derive the optimal mechanism. Rochet and Armstrong (1999) adopts a discrete type space to find a closed solution to a range of multi-dimension screening problem. Yet their solution does not apply to non-separable utility preference. One contribution of this paper is finding a credible criterion to set up an ordering of different types in a non-separable utility case.

As for the recent development in multi-dimensional screening, Rochet and Stole (2001) develop the direct method that the associated first order conditions needs to have a partial differential equation. Deneckere and Severinov (2009) study a screening problem where the type space is multi-dimensional, while the allocation space is one dimension. They transform the multi-dimensional screening problem to a one-dimensional optimal control problem, whose solution is governed by an ordinary differential equation. Dworczak and Zhang (2015) show that in a multidimensional screening problem, "implementable allocation rules correspond exactly to Walrasian equilibira of an economy" where consumers with quasilinear utility and unit demand are perceived as types.

In terms of analyzing the incentive compatible constraints, this paper closely related to two papers. The first is Armstrong (1996). Even though the type distribution does not enter into either the consumer's objective function or the firm's objective function, when Armstrong actually finds the optimal mechanism, the objective function is the sum of all the individuals' profit contribution, hence an expected value over the distribution of consumers' type. We can see later the result he obtains is strongly associated with the structure of the objective function. The screening mechanism in this two-sector model resembles the pattern of his proposition one: "a set of consumers of positive measure will not buy any goods at the optimum", means the low efficiency type will be excluded from the mechanism. But his setup is in a continuous environment, he hasnâĂŹt defined a concrete meaning of exclusion in a discrete setup. The second paper related is Rochet and Armstrong (1999). They model the multi-dimensional screening problem in an environment of non-linear pricing (or the government is regulating a multi-product monopoly). The preference is assumed to be separable across products (no cross-product price elasticity). The optimal solution to the problems is categorized into four different cases based on the correlation of the types' distribution over different dimensions. The intuition of the optimal solution is that the optimally allocation of the information rent to different types is based on the optimal participation of different types. This model has a complete solution. Yet in Rochet and Armstrong (1999), as they noticed, their method does not apply to non-separable preferences. Their "high type activity levels" achieve first best invariant of the type distributions result can be proved in a non-separable preference environment.

The point of emphasizing the composition of the objective function is to justify the method I adopt to solve the optimization problem in a multi-dimensional setting. Because the uncertainty about the type is based on the prior, the best the mechanism designer can do is to maximize over the expected value. Also combined with the special of BOT contracts: the two regulatory decisions should be combinatorial optimized. Hence based on the nature of the problem, I apply a Lagrangean decomposition method proposed by Kim (1986) and Baumann, Buchheim and Ilyina (2014) to consider the government's multi-dimensional screening problem of a BOT contract. The Lagrangean Decomposition is one method in Mean-Variance combinatorial optimization in robust discrete optimization problem.

2.3 Two-Sector Model with Two Different Reporitng Schemes

For a project seeking for construction and operation concessioners, I assume there are two regulatory agencies in the government making independent decisions. The construction department is responsible for construction decision of the project. The operation department is responsible for operation regulation and maintenance. The government lacks of the technology to construct but is able to operate on its own. The two regulatory agencies observe no decisions from the other but they share the same objective function.

During the whole concession process, they make independent decisions: they do not communicate and they do not observe the other department's decision. Let us take a note here: this is the key assumption that would affect the validity of the Lagrangean decompositions of the problem.

Two different reporting schemes correspond to the different ways of decomposition of the multi-dimensional screening problems for a single regulator. The first reporting setup is the private contractor should report both the costs of construction and operation to different departments, called full-report and the second setup is: the contractor will report different cost to different department, namely reporting construction cost to construction sector only and operation cost to operation sector only. The second reporting scheme is called complete separate report.

The complete separate reporting scheme refers to the decomposition idea of translating the multi-dimensional screening problem into multiple one-dimensional screening problems. The full reporting scheme refers to the decomposition idea of translating the multi-dimensional screening problem into screening the same problem over different subsets of the constraints. I prove the first decomposition is weakly dominated by the second one. And the first decomposition is not a valid one. I also use two different reporting schemes to check how the organizational hierarchy would affect the government's optimal contracts and what the legislation implication on the information disclosure resulting from such hierarchy.

Model Setup

The public project under concession has a life span n after construction. And it has a stable revenue flow during the operation stage. If the contractor is operating, the revenue from the operation stage is R; if the government is operating on its own, the revenue is s. There is no assumed relationship between R and s.

The government lacks the technology to build the project but is capable of operating. So the private contractor can be just hired to do construction or do both construction and operation. We refer the later case as Build-Operate-Transfer (BOT) contract. Since we assume the final ownership of such public project belongs to the government.

The private contractor under concession has two dimensions of private information: its construction cost F_i and its operation cost c^j, $(c^j; F_i) \in R_+ \times R_+$. In this two-sector model, there are two different regulation agencies: the construction regulator and the operation regulator. Both regulatory agencies ask the contractor's full private information on both dimensions.

In this two-sector model, we assume both sectors share the same objective function; therefore there is no strategic interaction between different regulation agencies and we assume there is no interaction between different government's regulation agencies.

Every regulatory agency has limited decision power and instruments to a certain part of the entire public good provision plan.

$$\sum_i \alpha_{ji}\{-T_i^j + [Q^j(n - t^j)s + (1 - Q^j)ns - O^j]\}P_i^j$$

At the operation stage, with probability Q^j, the government operates the project on its own for $n - t_i^j$ years (after the contractor's authorized phase expires) generating revenue s each year and asks for O^j; with probability $1 - Q^j$, the government is operating on its own for the project's entire life span. At the construction stage, the monetary transfer between the government and the contractor is T_i^j, with probability P_i^j, the contractor gets to construct such project. The contractor (c^j, F_i) is drawn from an arbitrary joint distribution with a prior of α_{ji}, where $\sum_i \sum_j \alpha_{ij} = 1$ and $i, j = H, L$. This means the contractor can be high efficiency or low efficiency on both dimensions accordingly.

The private contractor's utility function is:

$$P_i^j \{Q^j [t^j (R - c^j) + O^j] + T_i^j - F_i]\}$$

The contractor can get revenue R during the authorized operation phase, incurring a operational cost c^j. If the project is built by it of probability P_i^j, the contractor will also incur a construction cost F_i and the contractor will receive a total monetary transfer T_i^j from the construction department and O^j from the operation department.

Again, the projects here have their characteristics: first, no matter whether the contractor or the government operate, the project will generate a stable revenue during the whole operation process, either R or s. Second, the project itself will expire after n years.

Timing

- Construction sector announces mechanism $(P_i^j, T_i^j) \in [0,1] \times R$. P_i^j: the probability, contractor ji with construction cost F_i and operation cost c^j to do the construction work. $P_i^j \in [0,1]$. T_i^j: the monetary transfer paid by construction sector.

- The private contractor reports cost structure $[c^j, F_i] \in R_+ \times R_+$ or F_i

- Operation sector announces mechanism $(Q^j, t^j, O^j) \in [0,1] \times [0,n] \times R$. t^j: the length of the project is operating by the contractor; O^j: the monetary transfer that would be charged by the operation sector; Q^j: the probability that the contractor will get contract j $(t^j; T^j)$ at the operation stage

- The private contractor reports cost structure $[c^j, F_i]$ or c^j.

- If the public project is ever built, either a construction only or BOT contract is utilized, otherwise, there will be no such public project.

Different regulation agencies sequentially make their independent decisions without communications.

2.3.1 The Operation Sector's Problem

We solve the two-sector model using backward induction.

Without communication, the operation sector treats the construction cost as sunk. The operation regulator updates its beliefs of the type distribution about who would be selected to construct, then decides who to give the authorized operation phase.

Two different reporting schemes differ at the operation stage only in terms of the regulator's posterior beliefs, otherwise the pattern of the optimal solution is the same.

Since the construction cost is sunk, for the operation sector is adjusted to:

$$\sum_j \alpha^j [Q^j(n - t^j)s + (1 - Q^j)ns - O^j]$$

The type distribution α^j is the operation sector's speculation about types. It will be adjusted according to the Bayesian updating.

In the case of full reporting, these beliefs are:

$$\alpha^H = \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}} \quad \text{and} \quad \alpha^L = \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

In the case of complete separate reporting (decision-relevant information only), the operation regulator's beliefs are:

$$\alpha^H = \frac{\sum_i \alpha_{Hi}}{\sum_i \sum_j \alpha_{ji}} \quad \text{and} \quad \alpha^L = \frac{\sum_i \alpha_{Li}}{\sum_i \sum_j \alpha_{ji}}$$

Since the construction cost is sunk, in the operation the contractor's utility should be:

$$\{Q^j[t^j(R - c_j) + O^j] + T_{i*}^j - F_i\}P_{i*}^j$$

If there is no communication (observation) between different regulatory agencies' decision, such institutional setup as a rule of the game would make the construction decision sunk for the operation regulatory agency. Even though the operation department can make the contractor reported the construction cost, it does not have the instruments to affect the contractor's utility based on such information as sunk cost.

In this section, I only present the institutional setup of no communication between two regulatory agencies to justify the assertion that the decomposition of the two-dimensional screening problem to two one-dimensional screening problems is dominated.

We first see the operation agency's problem.

The operation sector's program is:

$$\max_{(\mathbf{t},\mathbf{O},\mathbf{Q})} \sum_{j\in\{H,L\}} \alpha^j [Q^j(n-t^j)s + (1-Q^j)n \cdot s - O^j]$$

$$s.t.$$

$$\forall j,k \in \{H,L\}$$

$$Q^j[t^j(R-c^j)+O^j] \geq Q^k[t^k(R-c^j)+O^k] \tag{IC}$$

$$Q^j[t^j(R-c^j)+O^j] \geq 0 \tag{IR}$$

$$0 \leq t^j \leq n$$

$$0 \leq Q^j \leq 1$$

The operation sector's problem can be characterized by two propositions: Proposition 4 shows that the optimal decisions for the operation sector is not random: treating construction cost as sunk cost, the operation regulator optimally extracts the relative operational efficiency through charging a franchise fee from the contractor and let the private contractor operate the entire life-span. The optimal decision is trading off the optimal rent paying and relative efficiency extraction. Proposition 5 shows how exactly the operation regulator extracts the relative operational efficiency in different environment. Again since the construction cost is sunk, the construction regulator's decision affects the operation regulator through affecting the operation regulator's beliefs on who would construct and enter its choice set.

Proposition 4. *The operation sector's decisions are not optimally randomized.*

Assume the operation sector randomly gives the low operation efficiency contractor a contract, by doing that the government is supposed to reduce the information rent paid to the high efficiency one. However, this would also reduce the government 's possibility of extract the relative operational efficiency from the low efficiency type. Since the optimal allocation of the operation decision is linear: either the government or the contractor operate, and the rent paying itself has been considered for the decision of allocation(we will let both high and low operational efficient type operate if and only if $R - s > c_L + \frac{\alpha^H \Delta c}{\alpha^L}$, therefore the randomization will have lost on the relative efficiency extraction which is bigger than the information rent paid to the contractor.

Proposition 5. *The optimal operational stage contract is characterized by:*

- *When the private contractor is much operational efficient satisfying $R - s > c_L + \frac{\alpha^H (c^L - c^H)}{\alpha^L}$, every type enters the operation stage operates.*

$$t^L = t^H = n$$

$$O^H = O^L = -n(R - c^L)$$

- *When only the high operational efficient type is relatively efficient than the government, satisfying $c_L + \frac{\alpha^H \Delta c}{\alpha^L} > R - s > c_H$, only high operationally efficient type(s) operate(s).*

$$t^L = 0$$

$$t^H = n$$

$$O^H = -n(R - c^H)$$

$$O^L = 0$$

- *When the government is relatively more operationally efficient than the contractor, satisfying $R - s < c_H$, the government operates on its own.*

$$t^L = t^H = 0$$

$$O^H = O^L = 0$$

Given the updated type distribution $\hat{\alpha}$, the operation sector can optimally extract the contractor's relative operational efficiency based on the updated beliefs. When even the low operational efficient type is relatively more efficient than the government (after considering rent-paying to the high type by including of the low types), then the operation sector should let all the types operate. Since this decision has already considered the information rent paid to the high type on the operation dimension, randomization will not play a role in the optimal decision. When the government's operation efficiency is in between the high and low operational efficient type, the government should only accept the high operational contractor to operate and exclude the low efficiency type. If the government is more efficient than the high efficiency type, the government should operate on its own.

The optimal operational decisions are based on taking the rent paying into consideration, how much relative operational efficiency can be extracted from the contractor. The operation department uses its own operational revenue s as criteria to distinguish the low operational type from the high type.

It is very important to notice that: since the construction cost is sunk (under the no communication between departments assumption), the contractor has to make sure it is making non-negative utility at the operation stage, which creates an inefficiency of the latter problem. The sunk cost results from the institutional assumption that there is no communication between the two different regulatory agencies.

2.3.2 The Construction Sector's Problem

Because the operation sector will update its beliefs about the type distribution accordingly to the construction probability, which is a decision made by the construction agency. Namely the optimal operation contract is a function of the optimal construction probability. That means the best way to extract the contractor's relative operational efficiency depends on who will be given the right to construct at the first place. For the construction sector, such decision shall be made by considering: first, what is fundamental value of the project $ns - F_i$, which is the utility for the government when it operates on its own and the contractor build the project. Under every circumstance, the project will be built only if this value is positive; second, the relative efficiency the operation sector can extract from the contractor and last, the rent paying on both dimensions.

The partial ownership at the operation stage will internalize both construction and operation decisions if the contractor's relative operational efficiency is high, however, It is still plausible that treating construction cost as sunk cost would eliminate certain flexibility of the construction sector. The operation sector will ask the contractor to pay a fee to get the operation right. If all the types are operating, the operation sector has to pay operational rent to high efficiency types. BOT will help the government reduce rent paying. We know that the (c^H, F_H) type is able to pretend anyone at either stage. When the operation dimension's information rent is higher, and when the chance of the contractor being an (c^L, F_L) is low, the government should be able to exclude the(c^L, F_L) type from the contract and pool for the rest types. In that case no one but the (c^H, F_H)type gets pure private information rent on the operation dimension. HL type will have to use the information rent on the operation dimension to remedy its inefficiency on the construction dimension, all three types will get the same contract as if they were (c^L, F_H).

The optimal construction contract is not only considering the rent paying on both dimensions, but the optimal consideration is also based on possible type conjecture. Depending on the ex ante distribution of the types, the government is deciding which type to be included in construction and the amount of the monetary transfers should be paid.

Since the continuation value at the operation stage is settled for the government, the construction decision has only to do with the whether the contractor i will bring the positive surplus to the government or not counting its optimal continuation value.

$$\sum_i \alpha_{ji}\{-T_i^j + [Q^j(n-t^j)s - O^j + (1-Q^j)ns]\}P_i^j$$

From proposition 4 we know, there is no randomization for the operation sector's decision, therefore Q^j is a zero-one decision.

And the contractor's utility will be:

$$[t^{j*}(R-c^j) + O^{j*} + T_i^j - F_i]P_i^j$$

The continuation utility of contractor ji is $t^j(R-c^j) + T^j$, which is determined optimality by the operation sector, which is a function of construction sector's optimal decisions.

The construction section's program:

$$\max_{(\mathbf{P},\mathbf{T})} \sum_{i\in\{H,L\}} \alpha^{ji}[-T_i + Q^j(n-t^j)s + (1-Q^j)n\cdot s - O^j]P_i$$

$$s.t.$$

$$\begin{aligned}
&\forall i, m \in \{H, L\} \\
[t^{j*}(R-c^j) + O^{j*} + T_i - F_i]P_i \geq& [t^{k*}(R-c^j) + O^{k*} - T_m - F_i]P_m && \text{(IC)} \\
[t^j(R-c^j) + O^j + T_i - F_i] \geq& 0 && \text{(IR)} \\
0 \leq& P_i \leq 1
\end{aligned}$$

All the details of the Kuhn-Tucker conditions will be offered in the appendix at the end of this section.

Here are some observations of the first order conditions:

First of all, by observing the first order condition of the participation probabilities for different types, we can see that their change in the operation sector's speculation would not affect the optimal choice of P_i^js. The first order condition of P_i^js say that the government can get $ns - T_i^j$ by only letting the contractor construct and operate on its own, and there is some continuation value if the government let the contractor with a higher relative operational efficiency operate. The government's optimal construction decision would depend on both values as well as the IR constraints for different types.

Secondly, the first order conditions about the monetary transfers T_i^js give very intuitive meaning of all the Lagrangian multipliers. Conditioning on type ji participating in the contract, all the effects associated with this type (IR, IC) are summed up to the type prior distribution α_{ji}.

Thirdly, the exclusion decision depends on the type's surplus contribution up to its prior and rent-paying comparison. For example, the exclusion decision about the (c^L, F_L) type is depending on comparison between the (c^L, F_L) type's contribution to the government $nd_L - F_L$ up to its prior α_{LL} and the rent has to pay to other types by including (c^L, F_L): the construction information rent to the (c^H, F_H) and (c^H, F_L) and the information rent paid to the (c^H, F_L) and (c^H, F_H) on the operation dimension. In every case, (c^H, F_H) can get information rent on every dimension if any.

Proposition 6. *When the prior probability that the type being low efficiency on both dimensions is very low (the relative efficiency the government can extract from the low type is small):* $\alpha_{LL} < \frac{n\Delta c}{nd^H - F_L}$ *and information rent on the operation dimension is higher:* $n\Delta c - \Delta F > 0$. *The optimal construction stage contract will be characterized by:*

- *The LL type will be excluded from the contract:*$P_H^H = P_H^L = P_L^H = 1; P_L^L = 0$
- *The monetary transfers paid to different types will be:* $T_H^j = F_H$;$T_L^H = F_H$

Proposition 7. *The construction regulation agency's optimal decision depends on the expected value of the project $\alpha_L(n\,s - F_L)$ after the rent concerns $\alpha_H(F_L - F_H)$, where $\alpha_i = \sum_j \alpha_{ji}$ with $i, j = H, L$:*

- *When the value of the project is large enough, $\alpha_L(n\,s - F_L) - \alpha_H(F_L - F_H) > 0$: all types are asked to construct, $P(c^j, F_i) = 1$; every type receives the same compensation: $T(c^j, F_i) = F_L$.*
- *When the value of the project is low, $\alpha_L(n\,s - F_L) - \alpha_H(F_L - F_H) < 0$, the low type(s) on the construction dimension are excluded from the contract, $P(c^j, F_L) = 0$, high constructional efficient type receive zero rent $T(c^j, F_H) = F_H, j = H, L$.*

In Proposition 6, the continuation value for the contractor is the operational rent. If the information rent is higher in the operation dimension and the prior probability that the contractor being a (c^L, F_L) type is low, then all types except (c^L, F_L) are doing BOT contract, (c^H, F_H) would want to pretend he is (c^L, F_H) and get the information rent on the operational dimension; however the (c^H, F_L) type needs to use its information rent on operation dimension to compensate its inefficiency to construct; and (c^L, F_H) achieves first best.

Exclusion first of all is depending on the rent paying and the final contribution of types' to the government, which is composed by the fundamental value of the project and the continuation from the contractor.

Optimal decisions about P_i^{j*} should be based on what essentially the type ji contractor can bring to the government. For sure the contractor should at least bring the government $ns - F_i$(the fundamental value), that is the value which the government has the contractor build the project, and operate on its own. And if the contractor is more efficient on the operation dimension, the government shall be able to extract this relative efficiency $(R - c^j - s)$ by having the contractor operate.

Also since the government will be able to extract the relative efficiency on operation dimension (proved by proposition 2) after rent-paying, and the high-efficiency type's utility is always coming from the rent. By including a low type, the government has to pay rent to the high one. Therefore the optimal decision about P_i^j will consider both fundamental value of the type; the continuation value of the type (the relative efficiency on the operation stage) and the rent paying on both dimensions.

Comparing the optimal contracts of two different reporting schemes (given by Proposition 6 and Proposition 7), the complete reporting scheme weakly dominates the partial reporting scheme. Because the construction is no able to distinguish the (c^L, F_L) and (c^L, F_H) hence when the project is very valuable, the contractor who is low efficient on both the construction and operation dimension is more likely to be hired to do BOT. The partial reporting scheme makes the construction regulatory agent's selection criterion coarser. In other words, the decomposition method of each agent solving one-dimensional screening problem is weakly dominated by the decomposition methods of solving the original problem in two different subsets. Later, when we talk about conditions in which the single regulator's problem can be decomposed into two regulatory agencies' problems, we refer to the full reporting scheme.

2.4 The Single Regulator's Model

The setup of the model will be exactly like the previous two-sector models. For a project that can generate a stable flow of revenues in the operation stage, the government lacks the construction technology and tries to find out what the best concession contract is. The general model is static and with only one principal. The government as the principal optimally chooses three instruments: the authorized phase t_{ij} and the participation probability P_{ij} and the monetary transfers T_{ij} and the contractor reports its type $(F_i; c^j)$

In this section, we will first present the government's program and then report several important observations of the first order conditions of our choice variables. The specific conditions will be provided at the appendix.

The government's program in the general mechanism is:

$$\max_{(\mathbf{P},\mathbf{t},\mathbf{T})} \sum_{i \in \{H,L\}} \alpha_{ji}[(n - t^j)s - T_{ji}]P_{ji}$$

$$s.t.$$

$$\forall i,, j, m, l \in \{H, L\}$$

$$[t_{ji}(R - c^j) + T_{ji} - F_i]P_{ji} \geq [t_{ml}(R - c^j) - T_{ml} - F_i]P_{ml} \tag{IC}$$

$$[t_{ji}(R - c^j) + T_{ji} - F_i] \geq 0 \tag{IR}$$

$$0 \leq P_{ji} \leq 1$$

A few important observations of the first order conditions can offer us some hints about the optimal contracts. Each type's effect for the government is up to its prior, therefore when considering extracting efficiency and paying rents, prior speculation is especially important for the government to achieve optimality. Namely exclusion of types will be part of the optimal contract, and this is closely associated with the previous attributes of the best choice of the instruments.

Proposition 8. *The construction decision is not random in the optimal contract.*

First of all, the optimal authorized phase can optimally extract the contractor's relative operational efficiency. The optimal construction decision will consider both dimensions of private information. Since both optimal allocation decisions are linear in the government's utility, instead of randomization, exclusion of certain type will be optimal. Exclusion of types depends on the prior of type distribution and the information rents paying on both dimensions. If the net surplus a certain type can generate is lower than the information rent paid to other type by inclusion of such type, this type should be excluded from the contract.

Second, different from the two-sector model, the construction cost is not sunk cost in this scenario. Therefore we see an extended proposition 6. If the information rent is higher on the construction dimension and the prior probability of (c^L, F_L) is very low, the government will exclude (c^L, F_L) type and treat the rest types as if they were all (c^H, F_L). In that case, (c^H, F_H) only get information rent on construction dimension, the (c^H, F_L) type get the first best contract and the (c^L, F_H) type uses information rent on the construction dimension to compensate the inefficiency on the operation dimension.

Proposition 9. *Under the condition all types can create positive utility for the government and relatively operational efficient. BOT contract can be used to reduce the rent paying by pooling and partial pooling (pooling after exclusion) for types according to the prior distribution and depending on rent paying on different dimensions, pooling conditions will differ:*

- $\Delta F > n\Delta c$
  - $\alpha_{LL} < \frac{n\Delta c}{nd^H - F_L}$: $P_{LL} = 0$, $P_{HH} = P_{HL} = P_{LH} = 1$.
  - $\alpha_{LL} > \frac{n\Delta c}{nd^H - F_L}$: $P_{ij} = 1$.
- $\Delta F < n\Delta c$
  - $\alpha_{LL} < \frac{\Delta F}{nd^L - F_H}$: $P_{LL} = 0$, $P_{HH} = P_{HL} = P_{LH} = 1$.
  - $\alpha_{LL} > \frac{\Delta F}{nd^L - F_H}$: $P_{ij} = 1$.

Since the contractor's actual utility will come from rent if any there. That means the high type would always have the incentive to mimic the low type. In other words, inclusion of the low type would mean a rent must be paid to the high type. When the prior that there is low possibility for (c^L, F_L) type, exclusion of such type would be optimal. Because the government only has to pay one-dimensional information rent to (c^H, F_H) type. When the prior of (c^L, F_L) is relatively high, the optimal thing to do is full pooling: treat everyone as if they were (c^L, F_L)L.

When the operational dimension information rent is high and exlucsion is proper. The optimal contract says every one except (c^L, F_L) is doing BOT contract: they build, they operate and they get the same contract as if they were all (c^L, F_H) type. In this case, (c^H, F_H) type gets the operational information rent; the (c^L, F_H) type's participation condition binds and for the (c^H, F_L) type is using his information rent on the operation dimension to compensate its inefficiency on the construction dimension. If the information rent on the construction dimension is higher, the government is partially pooling (c^H, F_H), (c^L, F_H) and (c^H, F_L) as if they were all (c^H, F_L) and the logic is similar.

When the (c^L, F_L) type is excluded, the intermediate type ((c^H, F_L) or (c^L, F_H)) would have to use rent owned on one dimension to compensate its inefficiency on the other dimension if both intermediate types are participating in the contracts. Because the government is only paying information rent on one dimension, and depending on different cases, such rent can cover the inefficiency on other(which is the other dimensioal information rent), both intermediate types would have the incentives to participate and truth telling (because of the pulling contract).

As long as a low type on both dimensions are included. The (c^H, F_H) type can choose the one with higher information rent to mimic. The logic is just as we analyzed before. One can infer that if the prior of the (c^H, F_H) type is sufficiently high, the government would exclude all types but the (c^H, F_H) type. So what we present here is just a part of the optimal contract, which gives an idea of how the optimal contract should be written:

Based on the prior distribution and information rent comparison on each dimension, the optimal contract is function of both.

Different from the two sector model, where the government would treat construction cost as a sunk cost, in the single regulator model, if the probability that the types being low construction is low and the construction information rent exceeds the relative operational efficiency, the government would let the types with high construction efficiency to do BOT. Even though here the government here is more capable of operation, the government is using partial ownership offered by BOT contract to reduce the rent paying.

2.5 Lagrangian Decomposition of The Single Regulator's Problem

Since we have proved that the decomposition of the single regulatorâĂŹs two-dimensional screening problem into two one-dimensional screening problems is not valid. In this section, I drop the assumption that there is no communication between two regulatory agencies (one regulatory agency does not observe the other's decision) to test a correct decomposition method. I show how the single regulator's problem can be validly decomposed to two regulatory agencies' problems. I provide sufficient and necessary conditions for such decomposition. I then show how the no-communication assumption obscures the validity of such decomposition.

Two independent regulatory agencies share the same objective function but each has limited authority over the entire BOT contract, hence the constraints set is separated by this presumption. Fixing one distribution of the types, the combinatorial optimization problem is well defined for the two regulatory agencies and truly decomposable. Without the concern of uncertainty, the optimal solution to the decomposed problem is also the optimal solution to the original problem. I utilize the theorems in Kim (1996) to prove this.

First, recall the single regulator's program is:

$$\max_{(\mathbf{P},\mathbf{t},\mathbf{T})} \sum_{i \in \{H,L\}} \alpha_{ji}[(n-t^j)s - T_{ji}]P_{ji}$$

$$s.t.$$

$$\forall i,, j, m, l \in \{H, L\}$$

$$[t_{ji}(R-c^j) + T_{ji} - F_i]P_{ji} \geq [t_{ml}(R-c^j) - T_{ml} - F_i]P_{ml} \tag{IC}$$

$$[t_{ji}(R-c^j) + T_{ji} - F_i]P_{ji} \geq 0 \tag{IR}$$

$$0 \leq P_{ji} \leq 1$$

The prior distribution of types is on two-dimensional uncertainty for the government, hence we can define such uncertainty as A: all possible distributions over the four types (c^j, F_i), where $i, j \in \{H, L\}$

Where Co is the convex hull of the feasible sets.

- The IC and IR determine the nature of the convex hull of the choice set.

- By using Kim (1986), we are able to see how single regulator's problem can be decomposed to problems with two different constraints sets. Hence may solve the problem easier. The setup of the two regulatory agencies itself may obscure the decomposition of the single regulator's problem. Hence inefficiency may arise.
- By using Baunmann (2014) (will be further explained in the second subsection) we will be able to treat the Lagrangean as a black box and separating the uncertainty part in the objective function from its decision parts. Hence a solution to the general regulator's optimization problem

2.5.1 Lagrangian Decomposition Under a Fixed Prior Distribution

Fix a certain realization of distribution α, we can decompose the above problem into the following:

$$\max_{(\mathbf{P},\mathbf{t},\mathbf{T})} \alpha^T f(P,t,T)$$

$$s.t.$$

$$(P,t,T) \in Co\{\mathbf{P},\mathbf{t},\mathbf{T}\}$$

$$\begin{aligned} &\max_{(\mathbf{P},\mathbf{T_c},)} \alpha^T f(P,t,T_c,T^o) \\ &s.t. \\ &IC, IR \\ &t = t* \\ &T^o = T^{o*} \end{aligned}$$

and

$$\max_{(\mathbf{t},\mathbf{T^o})} \alpha^T f(P, t, T_c, T^o)$$

$$s.t.$$

$$IR, IC$$

$$P = P*$$

$$T_c = T_{c*}$$

The above problem is equivalent to creating one identical copy of the vectors of decision variables, in using one of these copies in each set of constraints and in dualizing the condition(s) that they should be identical.

The Lagrangean decomposition dual is shown to be equivalent to the mathematical programming problem defined on the intersection of the convex hulls of the feasible solution sets of the corresponding blocks. The primal interpretation provides us with a necessary condition for testing whether stronger bounds than conventional Lagrangean bounds can be obtained.

This decomposition of the problems says that for a fixed type distribution of the contractor, if the single regulator knows how to optimally use the instruments to maximize its objective function subject to the constraint set, the two sector regulators of the same objective function can also optimally independently choose their complementary instruments with the restrictions on their own choice sets and treating each other's decision as optimally fixed.

Here, the necessity is trivial under complete information. For every optimal truthful outcome implemented by two separate mechanism designers, the general mechanism designer just implements the optimal sub-components of the separate regulator's choice. In other words, the complementary implementability of collaborative departments can be integrated by the implementability of one general mechanism designer.

The sufficiency of such decomposition needs more work. The problems come from the combination of the two separate regulators optimal decisions may not have proper counter part in the general mechanism designer's menu choice set. If such combination can always be well defined, the necessity stands, which requires specific requirement on the preferences.

I can prove (in an abstract context) the necessity stands when the contractor's preference is convex and compact in the regulators' instruments. The abstract and the proof are provided in the appendix.

With each regulatory agency observes the other agency's decisions and no uncertainty of the private contractor's cost structure preserves, different regulatory agencies with constrained control over the entire mediation plan can still optimally make the allocation decision through their independent instruments. The single regulator's problem is decomposed into two problems on the complementary subsets of original constraint set. Each regulator is not screening one-dimensional private information but screening two-dimensional private information on their own constraint set by treating the other regulator's decision as optimally fixed.

Such decomposing is optimizing in the intersection of the convex hulls of the two regulators' constraints sets. Given the rule of the game is of communication between two regulatory agencies, then we need find the equilibrium outcome of the game to implement the optimal menu proposed for the given game. Since the timing of the game is sequential, construction regulator makes decision first and then the operation regulator. Using backward induction, the equilibrium result of the game where two regulators optimizing over their sub-constraints sets and implement the same optimal menu in the single regulator's case.

2.5.2 Lagrangian Decomposition with Uncertainty

When the uncertainty presents, the rule of the game (whether there is communication between the two regulators) eventually determines the equilibrium outcome. If we were to find equilibria such that they implements the menus (mechanism), the optimality of implementable mechanisms are determined by the best equilibrium achieved based on the rules. When we assume a" two independent working agencies making complementary decisions without communication" rule, independent agencies make decisions based on the conjecture of the other agencies' decision. Such conjecture has to be updated based on Bayes rule. In Bayesian Nash equilibrium, each agency's decision is best responding to its belief (on the other agent's decision). How the uncertainty of the prior obscures the decomposition process would offer us a different perspective on solving a multidimensional screening problem under a combinatorial structure. Hence it also offers a perspective on how the information disclosure implied by the institutional setup affects the optimal contract design. Based on Baunmann (2014) we can separate the uncertainty of the objective function from its underlying combinatorial structure under certain conditions.

Now, we change α back to binary distributions over the two dimensions. By introducing the min operation, we will be able to find out the lower bound of the optimization value, and comparing that to the general Lagrangian relaxing.

$$\min_{\alpha \in \mathbf{A}} \max_{(\mathbf{P},\mathbf{t},\mathbf{T})} \alpha^T f(P,t,T)$$

$$s.t.$$

$$(P,t,T) \in \mathbf{P},\mathbf{t},\mathbf{T}$$

Given the discreteness of the multi-dimensional screening problem as well as the non-separable feature of the regulators' and the agent's utilities in the instruments. The single regulator's problem essentially is using the allocation instruments to first guarantee the fundamental value the government expected from the public project, then to share the extra operational efficiency of the private contractor with the consideration of the information rent paid on both dimensions. Since in this screening menu, if any low efficiency type on either dimension is included in the menu, the information rent has to be paid on such dimension. Therefore the decision on including the type in the contract or not depends on the value such type can create through the project; the information rent expected to be paid and the prior based on the conjecture of the game.

When there is no presumed institutional requirement on the communication of the regulatory agencies, actually the operation regulatory agency would not treat the construction agency's decision as a sunk cost. Under such game structure, the two regulatory agencies indeed work as a single regulator. The decomposition of the single regulator's Lagrangean is legitimate on the subsets of both the construction and the operation regulators' problems.

Given the setups here is eligible to separate the uncertainty and combinatorial structure of the optimization problem, we know from the last subsection that the maximization is actually is on the intersection of those two subsets. Such intersection fully contains the optimal decisions under different conditions. In the intersection, the operation regulatory agency can optimally extract the private contractor's relative operational efficiency. Recall, the government is able to operate, hence the operational instrument is used to extract the relative operational efficiency. In the intersection, the construction regulatory agency makes the decision on the construction based on the optimal extracted relative efficiency on the operational dimension, the rent paying on both dimensions and the true contribution for the project of each type for a given set of the environment parameters.

The uncertainty enters into the single regulator's objective as partitioning the regulator's menu into different contingent plans. Because for every given type distribution, the government knows how to optimally allocate the instruments hence to maximize the value of the project in consideration. For every combination of the environment and the prior distribution, the government has a contingent menu. The uncertainty is separated from the combinatorial structure of the optimization problem. For the single regulator's problem, the uncertainty is jointly distributed over the rectangle, which is formed by two binary distributions on two dimensions. This also explains why the decomposition I previously mentioned is invalid because the decomposition of each department treating a single dimensional screening problem is under an implicit assumption that the two dimensional private information is independently distributed and that would limit the optimal solutions.

To apply the theorem proposed by Baunmann (2014), the following can be showed.

- **P**, **t**, **T** is a vector space, subset of R^3
- IC and IR constraint set is a subset of R^3
- c is $\{\alpha_{ij}\}$
- The lagrangian decomposition of the program
- full correspondence

Given the with communication institutional setup, the uncertainty partially resolved by observation (communication), hence even with uncertainty, we can again prove that the single regulator's problem is truly decomposable into two sub-problems for two independent regulatory agencies with constrianed control over the entire mediation plan.

At the presence of uncertainty, the sufficiency of decomposition no longer only stands with nice properties on the contractor's preference. Given the presumption of no communication between two departments, the game structure changes. Even though two regulatory agencies have the same objective function, without communication, the operation regulatory agency has to update its beliefs about the optimal decision of the construction regulator. Not every construction regulatory agency's decision can be rationalized based on Bayesian updating, hence "non-rational" behavior will be treated as sunk cost, some optimal equilibrium of the single regulator may not be able to be justified by the barrier of communication in the two-sector model. Therefore the game structure defined by the institutional setup may obscure the decomposability of the single regulator's Lagrangean problem. Given an institutional setup with constraints, the optimal single regulator's plan sometimes cannot be implemented by two independent regulators.

The following two claims briefly describe what would happen under the information restriction implied by the institutional setup within the government and how that would affect decomposability of the single regulator's problem and further more offer simpler solution to a class of multi-dimensional screening problems that have combinatorial features.

Claim One: If the private contractor's utility is non-separable in two regulators' complementary instruments, for every given prior of type distribution, the two sector regulators can separately optimally truthfully implement optimal instruments only if given the game structure, the posterior can be updated according to Bayesian Updating

When the information rent on the construction dimension is higher than that on the operation dimension and when prior of the type being (c^L, F_L) is very low. Then the construction regulatory agency should exclude the (c^L, F_L) type and let all other types construct as if they have high efficiency on the construction dimension. Other types except (c^H, F_L) will efficiently construct the project but (c^H, F_L) will incur a cost $F_L - F_H$ at the construction stage. The operation regulatory agency can reasonably update its beliefs under this condition. Given that even the low operational efficient type has higher operational efficiency than the government, all types enter into the operation stage operate as if they were the low operational efficient type. All types but (c^L, F_H) earn the information rent on the operation dimension, but (c^H, F_L) use the information rent to reimburse the loss at the construction stage. The optimal contract is excluding (c^L, F_L) and pooling all other types as if they were (c^L, F_H). This menu can arise as Bayesian Nash equilibrium hence implementable by the two-sector decomposition with or without communication.

Claim Two: Given a zero probability event (no Bayesian updating is applicable), the "optimal " uses of the instruments are not truthfully implementable for the complementary regulators.

When the information rent on the construction dimension is lower than that on the operation dimension and the prior of type (c^L, F_L) is low. The optimal construction decision would either be excluding (c^L, F_L) type and treat every other type as if they have the low construction efficiency (recall, the case we talk about now is under full reporting scheme). The operation agency updated its belief and treat all types enter into the operation stage as if they had high operational efficiency. In such case (c^L, F_H) will not be able to participate in the no communication institutional setup. Because both the operation regulatory agency and the private contractor will treat the construction cost as sunk (as well as the information rent). No Bayesian equilibrium can arise to implement such menu.

2.6 Conclusion

When we confront a very specific situation in multi-dimensional screening such that the number of the available allocation instruments is the same as the dimensionalities of the private information. We might attempt to decompose the multi-dimensional screening problem into sub-problems that using each allocation instrument to screen one dimension of the private information. Yet, we can prove mathematically, this decomposition method is not valid. The right way to decompose a multi-dimensional screening problem is to decompose it into problems of optimizing the same objective function on complementary subsets of constraints.

Naturally, the two-sector setup of the BOT contract stipulation is one of the environments where the above combinatorial optimization method can be used to decompose the single regulator's optimization problem. I proved that under the "regulators with communication" institutional setup, the decomposition is valid where two independent regulatory agencies are optimizing the same objective function using their own allocation instrument only by treating the other department's allocation instrument as optimally fixed. But with institutional setup of no communication, other issues may arise again to obscure a valid decomposition. By separating the uncertainty of the regulatorâĂŹs (or regulators') objective function from its underlying combinatorial structure, we showed that decomposability also depends on the rules of the game: the institutional setup of the game. Using economics intuition, if an optimal menu can be implemented, we know that there is an equilibrium arising from the game and the equilibrium outcome implements the optimal menu. When there is no communication between two regulatory agencies, some of the best equilibrium outcomes of the single regulator have no counter part in the two-sector model. The optimal menu proposed by the single regulator cannot be implemented by independent regulators without communication.

CHAPTER 3

LIABILITY CONCERNS IN PUBLIC-PRIVATE PARTNERSHIPS

3.1 Introduction

A contract is defined as a written or spoken agreement, especially one concerning employment, sales, or tenancy that is intended to be enforceable by law. Contracts should provide creditable guideline for parties involved and propose voluntarily implementable mechanisms to address different parties' concerns and coordinate their interests. Analyzing institutional arrangements among parties for a certain contract objective is at the heart of research on contract theory. This paper focuses on how a certain Public-Private-Partnership (PPP) contract can serve as the proper institutional agreement between the government and private sectors to provide sufficient public goods with random revenues.

A successful application of PPP contracts in US is the Chesapeake Bay Forest Project. In order to address the environmental issue is Chesapeake Bay, the local government decides to focus more on land and wetland management in order to solve the pollution in the bay area but lack of the financial and personnel resources. The government then uses a two-phase procedure to select a partner to address the problem. First the state government is working with NGOs to seek funding to purchase the land from the owner of those lands and then the government is partnering with a private sector who is required to manage the land property according to environmental standards and the revenue from the project is shared by all parties involved. In order to attract private sectors to participate in such project, the government is promised to undertake whatever loss incurred in the first two years and the private sector is fully responsible to address any issue from then on. Fortunately for the project in Chesapeake Bay, it turns out the project can generate sufficient revenues. However, not every project contracts with PPP is able to face such revenue condition. If an adverse condition happens, how the government stipulates a PPP contract to induce the private sector to make proper effort. Inspired by this real life example, I present the following model to analyze the constrained optimal contract stipulation. I treat the above NGO and the private sector the government partners with in the second phase as one PPP entity and they specifically use a PPP contract -BOT. I model the risk allocation conditions in Chesapeake Bay as the government is facing different limited liability constraints. In such case, the government has to make up whatever loss incurred by the private sector in the first two years, which will create a limited liability constraint for the private sector's utility during operation. In this simple model, I assume this limited liability exists whenever the private contractor is chosen to operate. Then in order to design the optimal contract with one instrument- authorized phase, how should the government allocate it in order to encourage proper effort? Also because there is only instrument in this model which is bounded by 0 and the lifespan of the project n, therefore this would create another limited liability constraint for the government. The importance to distinguish the effect of these two different liability constraints is to offer clear defined choice set for the policy makers.

This paper answers one question: How will a constrained partnership between the government and the private sector give the right incentives to the later to perform well? The answer is not obvious for two brief reasons: first, conditioning on the possibility that the government gives the project to the private contractor, because of the randomness of the revenue in such potential partnership, the contractor may not be able to generate enough revenue to make up a possible loss already happened. Second, without any monetary transfers as instruments, the government who wants to build a project with certain life span has another layer of constraints in terms of limited liabilities. I will use the following one period model with moral hazard concern and liability constraints to see how the optimal contract will look like under all those limitations.

3.2 Literature Review

Iowsa and Martimort (2014) shows that when the state is verifiable, the completeness of the contract will reduce the risk premium paid to the contractor, and the private contractor should be fully insured of the exogenous shocks that are beyond the contractor's control. And when the state is not verifiable, the demand risk is more likely transferred to the contractor to prevent misreporting. However, the optimal contract design requires the comparison between the cost to make fully contingent contract and the risk premium paid to the contractor because those will be different across projects and countries. With better institutions, the government is more likely to design complete contract, otherwise, when the cost is high and the institution is weak, at the contract design stage, the government will leave space for incomplete contract which may lead to corruption in the future. They show that corruption may also have a role to play at the ex-ante stage when parties decide how detailed their agreements should be. Weak institutions prone to corruption may also be associated with incomplete deals. Because those incomplete deals are also those most likely to be renegotiated, the impact of corruption on contract design and economic performances is likely to be even more significant than suggested by the earlier literature that focused only its ex post role. In this paper, instead of considering with a risk-averse private sector, I consider a risk neural one but with two different limited liabilities. The first limited liability stems from the contractor's effective utility has to be above certain level during operation. This is essentially an ex post participation constraint. The second liability concern comes from the only instrument the government could use. The minimum of the instrument the government would use is to deny the operation of the contractor. If the government ever wants to "punish" the contractor, it is limited by this lower bound. And the lifespan of the project is n, and the government is sharing the revenue with the contractor, therefore, the government wants to reward the contractor, it is limited by this upper bound as well. Therefore, this research studies how these two different limited liabilities will give rise to different inefficiency.

In the line of models with moral hazard of limited liabilities concerns, Poblete and Spulber (2009) offer necessary and sufficient condition in which the optimal contract takes the form of debt, which broadens Innes (1989)'s optimal contract application. They ask for two conditions: the distribution of shocks has monotone hazard rate and the shock and the agent's effort satisfy single crossing. This research relates the analysis of moral hazard and adverse selection that offers more insight. However, in order for the optimal contract to be debt, there should be monetary transfers used as the instrument. It will be interesting to see how the optimal contract looks like if the government has no monetary transfer, in the sense that the optimal contract can only take the form of share-cropping, what the efficiency loss is because of the contract form and distinguish it from the efficiency lost from the limited liability. I assume in this model the probability of the high outcome happens is an increasing concave function of the effort and therefore satisfy Poblete and Spulber's assumption.

Lawarree and Audenrode(1999) distinguish how the limited liabilities on transfer and on the contractor's utility in adverse selection model (Sappington 1983) and in moral hazard model. They showed those two constraints do not yield equivalent results in moral hazard condition as in adverse selection condition. The constraints on the utility are stronger because the optimal contract constrained by liabilities on utility is always Pareto dominated by that constrained by liabilities on transfers. It is sensible to distinguish the liability on the utility and the transfers, because distinguishing different constraint's effect would give policy maker a clear set of choice and their consequences.

3.3 One-time Interaction Model with Limited Liabilities

I first present a one-short model, in which the government has liability constraints and using Build-Operate-Transfer (BOT) as the public-private-partnership contract to work with a private contractor. BOT has the following contracting meaning: the private sector is introduced to do the construction because the government lacks the technology to do it on its own. Then after completion of the project, the private sector is authorized to operate the project for a certain period to compensate the cost both in the construction and operation stage. After the authorized phase for operation, the private contractor should give the project back to the government.

This model shows when the private contractor can make an effort to affect the project's revenue in the operation stage, the mechanism the government will use to encourage the private contractor to make proper effort is "punish when the revenue observed is bad; reward when the revenue is good", because the effort is not observable but revenue at the operation stage as an outcome is. We also found that that the liability weakly increase the difference between the government's punish-reward mechanism.

Model Setup

The government has a project seeking for a concessionaire. The government has one instrument: operation phase t authorized to the private contractor to operate in order to compensate the construction and operation cost and possible cost of the effort.

The project's revenue is random and depends on the effort made by the contractor at the construction stage. Assume once the contractor decides an effort level, the revenue stream will be steady during the entire operation phase.

Assume an effort e is going to lead to an revenue R_m and $R_m \in \{R_H, R_L\}$

Effort profile e is continuous on $[\underline{e}, \bar{e}]$.

Assume the probability of the revenue being high is $q(e)$, which is increasing and concave.

Define a feasible set $\{e|(n-t_H)R_H - (n-t_L)R_L > 0\}$

Based on two different outcome, the government will have two different payment schedule: $t_H; t_L$

Players

One government; one contractor

Payoff

The government's payoff:

$$q(e)(n-t_H)R_H + (1-q(e))(n-t_L)R_L$$

$$= (n - t_L)R_L + q(e)n(R_H - R_L) - q(e)(t_H R_H - t_L R_L)$$

The contractor's payoff:

$$q(e)t_H(R_H - c) + (1 - q(e))t_L(R_L - c) - F - e$$

$$= t_L(R_L - c) + q(e)(t_H R_H - t_L R_L) - F - e$$

The private contractor will have to incur both the construction cost F and the operation cost c, if ever operated.

Timing

- The government announces mechanism (t_H, t_L) for different outcomes observed and commit to it.
- The contractor makes effort and incurs construction cost
- Outcome observed either as R_H or R_L.
- The contract realized, and contractor incurs operation cost if ever operated.

3.3.1 Observable Effort

When the outcome is observable and without concerning the liabilities, the government would always want to encourage an effort to make a better project in order to generate more revenue in the operation stage. Therefore when the low outcome is observed, the government will punish the contractor by giving zero length of the project to the contractor to operate. I call the effort encouraged by the government under this condition the first best effort.

When the outcome is observable, but there are still limitations of liabilities both on the transfers the government can use and on the contractor's utility during the operation stage the government has to concern. I use $0 \leq t < n$ to represent the limited liability on the government's transfers and use $t(R - c) \geq \underline{w}$ to represent the liability concern on the contractor's utility. On one hand, because there is no monetary transfers in this

BOT contract, so the only instrument the government can use is the operation phase the government can grant the contractor to operate and this instrument is limited by the lifespan of the project itself which in total will be n years. On the other hand, since the government has to compensate the contractor's construction cost and operation cost, the government has to concern that the operation revenue during the contractor's operation phase has to be a certain level which is captured by $t(R-c) > \underline{w}$. This type of limited liability can also be considered as the ex post participation constraint. The result shows that the "first best" with the government's liability constraint only on the transfer -operation phase coincides with the first best without any constraint. The "first best " constrained by the contractor's utility has to be a certain level during operation stage yields lower efforts and therefore lower utility for the government.

Since the government can observe the contractor's effort, with full information and effort is observable, the government's problem is:

$$\max_{t_H;t_L;e} q(e)(n-t_H)R_H + (1-q(e))(n-t_L)R_L$$

$$s.t$$

$$q(e)t_H(R_H-c) + (1-q(e))t_L(R_L-c) - F - e \geq 0$$

$$t_H(R_H-c) \geq t_L(R_L-c)$$

$$0 \leq t_L < n$$

$$0 \leq t_H < n$$

The first constraint is the contractor's participation constraint. The second constraint is the monotone requirement, and the very last two are constraints on the transfers. Therefore ignoring the constraints on transfers first, the government would choose the instruments that bind the contractor's IR constraint.

$$q(e)[(t_H R_H - t_L R_L) - (t_H - t_L)c] + t_L(R_L - c) = e + F$$

Bring the above equation into the government's objective function, the government will get $q(e^*)[n(R_H - R_L) - (t_H - t_L)c] + nR_L - t_L c - F - e^*$. The government is expecting to reimburse the contractor as if the revenue observed is the low revenue case, and get the revenue difference with probability $q(e^*)$

Without the concern of the limited liability of the private contractor's effective utility at the operation stage, the government announces a contract $(t_H = \frac{F+e^*}{q(e^*)(R_H-c)}; t_L = 0)$ which induce an effort e satisfies $q'(e) = \frac{\frac{R_H}{R_H-c}}{n(R_H-R_L)}$ as the first best effort. However, under this circumstance, the liability constraint of the transfer when the observed outcome is low is actually binding.

With such concerns we have two more limited liability constraints to consider. We are able to ignore the liability constraint for the contractor's utility when the observed outcome is high, because the liability constraint for the contractor's utility when the observed outcome is low and the monotonicity constraint imply it. We can also conclude: it is never rational for the government to give the contractor the entire life-span without charging any fee upfront (recall, there is no monetary transfer in this model), because if the government does so, the government receives zero utility.

If the liability constraint on the contractor's effective utility at the operation stage binds,

$$q'(e^*) = \frac{\frac{R_H}{R_H-c}}{n(R_H - R_L) + (\frac{R_L}{R_L-c} - \frac{R_H}{R_H-c})\underline{w}}$$

Compared to the first best effort, with such limited liability constraint binding, the effort can be induced by the government is less. Therefore will lead to lower utility for the government.

Summary

But there are three important observations from the observable case:

(1) The government does not allow the private contractor to operate when the outcome observed is low. Therefore the pattern of the optimal contract in the observable case is always "reward when the outcome is high and punish when the outcome is low"

(2) When the limited liability constraint on the transfers (authorized phase) is binding, the government's utility would not change. The effort could be induced under this condition coincides with the first best effort. The intuition stems from the pattern of contract the government used to induce the first best effort. Because the government will reward the contractor when the observed the outcome is high and punish the contractor when the observed outcome is low. In this case, the worst punishment the government can use is set the only instrument it could use to be zero.

(3) When the limited liability on the contractor's utility during operation stage is binding, the government can induce less effort compared to the first best effort. The intuition is that the limited liability on the effective utility constrains the punishment of the government; therefore the less effort can be induced by the government's contracts.

(4) As the limited liability on the contractor's effective utility during the operation stage gets close to zero, the effort the government is able to induce gets close to the first best effort. When limited liability is zero, the effort can be induced by the government coincides with the first best effort.

3.3.2 Unobservable Effort

When the effort is observable, the government is using a "Carrot-Stick" contract to induce the first best effort. The goal for the contractor is therefore to maximize the value of the project and then insure its participation value.

When the effort is no longer observable, the contractor's effort is no longer focusing on creating values of the project for the entire life-span ($q'(e) = \frac{\frac{R_H}{R_H - c}}{n(R_H - R_L)}$) but only focus on what it can get from the allocated authorized phases under different conditions. In the effort observable case without the limited liability on the contractor's effective utility during the operation stage, the government only needs to worry about the contractor's ex ante participation constraint. Without observing the effort, how the government allocates the authorized phase will automatically create a limited liability on the contractor's effective utility during operation stage (showed by Lemma 1). Therefore, we can show in this very special case, different from the general conclusion in moral hazard models: with risk neutral agent, the first best effort is still achievable even when the effort is not observable. In this model, the first best effort is not achievable with unobservable effort.

Lemma 1 through Lemma 4 characterized the problem the government faces when the efforts are not observed with both kinds of limited liability constraints. Lemma 5 characterizes three possible conditions when there is a liability constraint. Proposition 1 states the first important result of the paper, the BOT contract with no monetary transfers has an intrinsic contract form flaw resulting from the limited liability constraint, and the first best effort is not achievable in this case. Proposition 2 through Proposition 4 characterizes the solution to the government's problem when the effort is not observable.

With the effort is unobservable, the government's problem is:

$$\max_{t_H;t_L} q(e)(n - t_H)R_H + (1 - q(e))(n - t_L)R_L$$

$$s.t$$

$$q(e)t_H(R_H - c) + (1 - q(e))t_L(R_L - c) - F - e \geq 0 \tag{IR}$$

$$e \in argmax\ q(e)t_H(R_H - c) + (1 - q(e))t_L(R_L - c) - F - e \tag{IC}$$

$$t_L(R_L - c) \geq \underline{w} \tag{LCU1}$$

$$t_H(R_H - c) \geq \underline{w} \tag{LCU2}$$

$$0 \leq t_L < n \tag{LCI1}$$

$$0 \leq t_H < n \tag{LCI2}$$

Lemma 1. *The individual rational constraint for the contractor can be written as* $t_L(R_L - c) \geq F + e - \frac{q(e)}{q'(e)}$.

We can conclude from the individual rational constraint can be written as a function of the contractor's effort.

Since $q(e)$ is a probability and concave function in e, then we can conclude from the above lemma that if the low outcome is ever observed, by getting the transfer from the government as t_L, the contractor is not able to completely make up the construction, operation and effort cost.

Proof.

- The second best e^* is given by the first order condition: $q'(e)[t_H(R_H - c) - t_L(R_L - c)] = 1$.
- Recall, the individual rational condition for the agent is: $q(e)t_H(R_H - c) + (1 - q(e))t_L(R_L - c) - F - e \geq 0$
- According to the incentive condition above: $t_H(R_H - c) = t_L(R_L - c) + \frac{1}{q'(e)}$
- Therefore the IR can be written as: $t_L(R_L - c) \geq F + e - \frac{q(e)}{q'(e)}$.

□

Lemma 2. *The lower bound of the government's reimbursement when the outcome observed is low* $F + e - \frac{q(e)}{q'(e)}$ *is a decreasing function in e.*

The individual rational constraint tells us that if the government wants the project to be ever built, then the government needs to reimburse the contractor with a certain level determined by the contractor's cost structure and effort on the quality of the project.

The above lemma tells us when the low outcome R_L is observed, the reimbursement the government gives to the contractor is decreasing in the contractor's effort.

When the low outcome is ever observed, the reimbursement given to the contractor is not sufficient enough to make up all the cost, and such reimbursement is decreasing in the effort. Given the fact if the contractor is working hard, a high outcome is more likely to be observed; the government will punish the contractor if a low outcome is ever observed.

Proof.

- The derivative with respect to e of $F + e - \frac{q(e)}{q'(e)}$ equals to $1 - \frac{q'(e)^2 - q'(e)q''(e)}{q'(e)^2}$
- $\frac{d[F+e-\frac{q(e)}{q'(e)}]}{de} = \frac{q(e)q''(e)}{q'(e)^2}$
- $q(e)$is a probability and a concave function of e, therefore $q(e) > 0$ and $q''(e) < 0$
- $\frac{d[F+e-\frac{q(e)}{q'(e)}]}{de} = \frac{q(e)q''(e)}{q'(e)^2} < 0$

□

Lemma 3. *The reimbursement for the high outcome R_H is an increasing function of e.*

Proof.

- $t_H(R_H - c) = t_L(R_L - c) + \frac{1}{q'(e)}$ according to the incentive constraint for the contractor and such incentive constraint is ex ante.
- According to the (IR), $t_L(R_L - c) \geq F + e - \frac{q(e)}{q'(e)}$
- Therefore, $t_H(R_H - c) \geq F + e - \frac{q(e)}{q'(e)} + \frac{1}{q'(e)}$
- $t_H(R_H - c) \geq F + e + \frac{1-q(e)}{q'(e)}$
- The derivative of function $F + e + \frac{1-q(e)}{q'(e)}$ is $2 - \frac{q''(e)(1-q(e))}{q'(e)^2}$, which is increasing in e.

□

Lemma 4. *The government's problem can be written as:*

$$\max_{\mathbf{t}} \; (n - t_L)R_L + q(e)[n(R_H - R_L) - (t_H - t_L)c - t_L(R_L - c)] - \frac{q(e)}{q'(e)}$$

$$s.t.$$

$$t_L(R_L - c) \geq max\{F + e - \frac{q(e)}{q'(e)}; \underline{w}\} \tag{IR}$$

$$t_H(R_H - c) = t_L(R_L - c) + \frac{1}{q'(e)} \tag{IC}$$

$$0 \leq t_L < n \tag{LLC1}$$

$$0 \leq t_H < n \tag{LLI2}$$

Proof.

- Since the government's utility is:

$$q(e)(n - t_H)R_H + (1 - q(e))(n - t_L)R_L$$

$$= q(e)nR_H - q(e)t_H(R_H - c) - q(e)t_H c + (1 - q(e))(n - t_L)R_L$$

- Bring IC to the government's utility function

$$(n - t_L)R_L + q(e)(nR_H - t_H c - nR_L + t_L c) - \frac{q(e)}{q'(e)} - q(e)t_L(R_L - c)$$

$$= (n - t_L)R_L + q(e)(n(R_H - R_L) - (t_H - t_L)c - t_L(R_L - c)) - \frac{q(e)}{q'(e)}$$

- According to the IC and IR constraint: $t_L(R_L - c) \geq F + e - \frac{q(e)}{q'(e)}$

- Then LCU1 can be implied by IR and LCU2 can be implied by considering IR and IC together.

- Observe the objective function: the government's objective is not only a function of the allocation (t_L) when the worse outcome is observed R_L and the possible effort level (e), but also an incentive created by different allocations $q(e)(t_H - t_L)c$.

- The liability constraint is $\underline{w}$, which can be perceived as an ex post utility the contractor needs to obtain during the operation stage.

- Therefore we can organize the government's program like the above.

□

Proposition 10. *When the limited liability constraint is irrelevant, the effort in the observable case is not achievable in the case where the effort is not observable and the limited liability on the contractor's effective utility during the operation stage is irrelevant.*

In the classical moral hazard environment, if the manager is risk neutral, the principal should be able to encourage the manager to exert first best effort by asking a fixed fee and selling the project to the contractor. In such case, the contractor would have the incentive to make the first best effort.

This proposition discloses the intrinsic contract form flaw of BOT contract with no monetary transfers for the first best effort is not achievable when the effort is unobservable. This flaw results from the limited liability constraints on the contractor's effective utility during the operation stage. When the effort is observable, the government is using "carrot-stick" contract to induce the first best contract. Combined with the limitation of using one transfer, the government is neither able to sell the project to the contractor, namely give a whole lifespan to the contractor, nor the government can punish the contractor harder than give it zero year to operate, therefore the first best effort is not achievable even when the contractor is risk neutral.

In this model, the first best effort is trying to gain the revenue difference when the outcomes differ focusing on the lifetime of the project. The allocation of the operation stage between the government and the contractor is used to compensate the contractor's operation, construction cost and effort cost. When there is no liability concerns for the contractor, depending on how much the government would like to compensate the contractor when the outcome is low, as such compensation increases, the difference between the compensation systems for different outcomes decreases. The first best happens where the low outcomes compensation achieves zero.

When the government is no longer able to observe the contractor's effort. Such first best effort is no longer achievable, since the contractor will only make using the benefit-cost analysis based on the authorized phase of the operation stage rather than the lifetime revenue of the project. Therefore liability on the contractor utility of profit during the operation phase is part of the participation concern. Based on the form of the individual rational constraint.

Proof. When $F + e - \frac{q(e)}{q'(e)} > \underline{w}$, therefore according to the result in the observable case, when limited liability on the contractor's effective utility is not zero, then the government can only able to induce a lower effort than the first best level.

- The first best effort e^* satisfies $q'(e^*) = \frac{\frac{R_H}{R_H - c}}{n(R_H - R_L)}$.
- If the government was able to induce the first best effort, e^* should satisfy with the (IC) contraint :$q^{'}(e^*)[n(R_H - R_L) - c(t_H - t_L) - t_L(R_L - c)] = 1 - \frac{q(e^*)q^{''}(e^*)}{q^{'}(e^*)}$
- Therefore the following condition must hold: $X = 1 - q(e^*)q^{''}(e^*)n(R_H - R_L)$, where X is clearly less than one with comparatively small operational cost.
- According to the model setup we know that $R_H > R_L$; and the $q(e)$ is a concave function, therefore $q'(e) > 0$ and $q''(e) < 0$ therefore we have the following
- Which according to the current model setup is not possible, therefore a contradiction.
- Therefore we conclude that the observable effort is not ever achievable.

$\square$

Corollary 1. *The second best effort is less than that of the observable case because different reimbursement (t_H, t_L) creates incentives for the contractor to shirk when the effort is unobervable.*

Proof.
- According to the IC constraint: $t_H(R_H - c) = t_L(R_L - c) + \frac{1}{q^{'}(e)}$

- The government's expected utility is:$q(e)(n-t_H)R_H+(1-q(e))(n-t_L)R_L = q(e)nR_H + (1-q(e))nR_L - (1-q(e))t_LR_L - q(e)t_HR_H$

- The objective function can be written as: $q(e)[n(R_H-R_L)-(t_H-t_L)c-t_L(R_L-c)]+(n-t_L)R_L-\frac{q(e)}{q'(e)}$

- The second best effort the government can enforce is :$q^{'}(e)[n(R_H-R_L)-c(t_H-t_L)-t_L(R_L-c)] = 1-\frac{q(e)q^{''}(e)}{q^{'}(e)}$

- Recall, the first best effort is determined by $q'(e^*) = \frac{\frac{R_H}{R_H-c}}{n(R_H-R_L)}$. The marginal probability $q'(e)$ to get the extra revenue $n(R_H-R_L)$ should be equal to the marginal effort to make that happen.

- Therefore for a pooling schedule, the second best effort the government can enforce is determined by $q^{'}(e)[n(R_H-R_L)-c(t_H-t_L)-t_L(R_L-c)] = 1-\frac{q(e)q^{''}(e)}{q^{'}(e)}$, hence with a comparatively small operational cost, an inefficiency would be created in terms of lower effort due to the marginal benefit of making the effort is less and the marginal cost of making an effort is higher.

$\square$

Lemma 5. *We can see from* (IR)*, the liability constraint creates three different conditions when the government does not barely want to implement the least effort.*

- When the liability constraint is not binding: $F+e-\frac{q(e)}{q^{'}(e)} > \underline{w}$

$$t_L = \frac{F+e^*-\frac{q(e^*)}{q^{'}(e)}}{(R_L-c)}$$

$$t_H(R_H-c) = t_L(R_L-c)+\frac{1}{q^{'}(e)}$$

Those three equations would solve for (t_L, t_H, e^*)

- When the liability constraint is binding: $F + e - \frac{q(e)}{q'(e)} < \underline{w}$

$$
\begin{aligned}
e^{**} \in argmax\ q(e)nR_H + (1-q(e))nR_L - t_L R_L - \frac{q(e)}{q'(e)} - q(e)(t_H - t_L)c \\
t_L = \frac{\underline{w}}{R_L - c} \\
t_H(R_H - c) = \underline{w} + \frac{1}{q'(e)}
\end{aligned}
$$

Those three equations would solve for (t_L, t_H, e^{**})

- When the liability conditions is irrelevant, $F + e - \frac{q(e)}{q'(e)} = \underline{w}$

The analysis of the above follows.

Proposition 11. *If the liability constraint is not binding, then $t_H - t_L$ is a decreasing function of e when $\frac{\frac{q''(e)}{q'(e)^2}[q(e)(R_H-R_L)-(R_L-c)]-(R_H-R_L)}{(R_H-c)(R_L-c)} < 0$, if otherwise, $t_H - t_L$ is an increasing function of e.*

When the effort is observable, the first best does not depend on the allocation transfers but on the entire lifetime of the project. Therefore the allocation transfers will only be used to make the contractor participated in the contract. In such sense, as there are continuous ways that the government could make the reimbursement plans for each outcome, the individual rational constraint is always binding for the contractor. As the government wants to increase the reimbursement when the outcome is low, the government would have to reduce the reimbursement when the outcome is high.

When the effort is no longer observable, the government needs to use proper reimbursement under different outcomes to induce proper effort. The revenue in the allowed operation stage is the only concern for the contractor. Therefore the government directly relate the reimbursement with the effort is a proper way to encourage effort. The above proposition tells us, if the revenue difference under difference is high enough, the reimbursement needed for different outcomes has smaller differences as the effort level increases. As the revenue stream even under low outcome is high enough, the above difference gets larger as the effort level increases. Proof:

When the liability constraint is not binding, therefore

$$\begin{aligned} t_L(R_L - c) &= F + e^* - \frac{q(e^*)}{q^{'}(e)} \\ t_H(R_H - c) &= F + e^* + \frac{1 - q(e^*)}{q^{'}(e)} \\ t_H - t_L &= \frac{\frac{1}{q^{'}(e)}(R_L - c) - (F + e^* - \frac{q(e^*)}{q^{'}(e)})(R_H - R_L)}{(R_H - c)(R_L - c)} \\ \frac{d(t_H - t_L)}{de} &= \frac{\frac{q^{''}(e)}{q^{'}(e)^2}[q(e)(R_H - R_L) - (R_L - c)] - (R_H - R_L)}{(R_H - c)(R_L - c)} \end{aligned}$$

If the difference of different reimbursement is an decreasing function of e, therefore more effort will cover more operational cost in terms of $-(t_H - t_L)c$ as a marginal gain with the (marginal) probability $q'(e)$. Otherwise, the difference would have a marginal cost.

According to the first order condition from the (IC), we could reorganized it and compared it to the first best case.

q'(e)[n(R$_H - R_L) - (t_H - t_L)c] = 1 - \frac{q^{''}(e)q(e)}{q'(e)^2}$

q'(e)[n(R$_H - R_L) + \frac{q^{''}(e)q(e)}{q'(e)^3} - (t_H - t_L)c] = 1$

Recall, the first best effort is offered by:

$$q'(e)[n(R_H - R_L)] = \frac{R_H}{R_H - c}$$

The first best effort is made to make the marginal gain from making the effort just equal to the marginal cost of the effort if the contractor is invited to operate. There will be no cost concerns for the optimal effort making. Therefore the effort will be made just worthwhile for the revenue difference. The cost of the contractor will totally be reimbursed from the authorized phase when the effort is observable.

When the effort is not observable, the difference of the reimbursement ways for different outcomes would first create incentives for the contractor to exert more (or less) effort which characterized by the term $\frac{q''(e)q(e)}{q'(e)^3}$, that is a gain by the government. However, different reimbursement ways when different outcomes are observed would also create cost in terms of the cost structure because now the effort is no longer observable, and the reimbursement ways should consider the cost structure as well.

Corollary 2. *If the revenue difference $R_H - R_L$ is large, then the optimal reimbursement $t_H - t_L$is decreasing in the effort*

Corollary 3. *If the fundamental value from the project $R_L - c$ is large, then the optimal reimbursement is increasing in the effort*

Proposition 12. *If the liability constraint is bind, then $t_H - t_L$ is an increasing function of e.*

Proof.

$$\begin{aligned}
t_L(R_L - c) &= \underline{w} \\
t_H(R_H - c) &= \underline{w} + \frac{1}{q'(e)} \\
t_H - t_L &= \frac{\frac{1}{q'(e)}(R_L - c) - \underline{w}(R_H - R_L)}{(R_H - c)(R_L - c)} \\
\frac{d(t_H - t_L)}{de} &= \frac{-q''(e)}{q'(e)^2(R_H - c)} > 0
\end{aligned}$$

That means though the different reimbursement would create incentives to work harder yet would contribute negatively on the margin as an operational cost rent.

□

Proposition 13. *The second best effort exerted on when the liability constraint is binding is generally less than that when the liability constraint is not binding.*

Proof. According to the first order condition:

$$q'(e)[n(R_H - R_L) - (t_H - t_L)c] = 1 - \frac{q''(e)q(e)}{q'(e)^2}$$

Therefore based on the fact that the difference created by a binding or non-binding constraint would only affect the term $q'(e)(t_H - t_L)c$, therefore based on proposition 1 and proposition 2, the government would want to implement higher effort in general if the liability constraint is not binding.

□

3.4 Summary

The difference of the government's reimbursement schedule creates the incentive for the contractor to exert effort and repay the costs but the effort induced is less than the first best effort level. Because the fact that the contractor only looks at the revenue in the operation stage would automatically create an ex post constraint in terms of limited liabilities on the private contractor's effective utility during the operation stage. The limited liability on the effective utility is stronger than the limited liability on the transfers in the sense that punishing the contractor by not allowing the contractor not to operate is now even not a choice.

In this model, the pattern of the government's reimbursement schedule for implementing second best effort is "rewarding the contractor when the outcome is high, punishing the contractor when the outcome is low ." Even though complete shut down the contractor's chance to operate is not available, however, the government could still punish the contractor by only giving enough cost reimbursement if the low outcome is observed. And reward the contractor more than its true cost when the high outcome is observed. Because the contractor's participation constraint is ex ante, therefore satisfied. However, because the punishment is limited, the incentives are limited.

When the liability constraint is not binding, the second best effort the government can implement tends to be higher compared to the case when the liability constraint is binding.

The government is able to satisfy the individual rational constraint ex ante.

3.5 A Numerical Example

Assume the probability function of effort is $q(e) = \sqrt{e}$ and e is choosing in the range $[0, 1]$. Therefore, $q'(e) = \frac{1}{2}e^{\frac{-1}{2}}$ and $q''(e) = \frac{-1}{4}e^{\frac{-3}{2}}$

According to Proposition 1, the optimal $t_H - t_L$ is always decreasing.

Assume $R_L = 1.5$ and $R_H \in [1.5, 4.0]$, $c \in [0.05, 0.10]$, $F \in [1.5, 3.5]$

The determinant term in this numerical example is $\sqrt{e}(R_L - c) - (R_H - R_L)$

The following observation is base on the liability constraint is not binding.

Observation 1

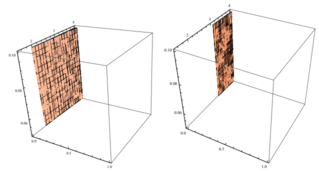
As the length of the project increases, the second best effort is increasing. Then values are set to be: $R_L = 1.5$ and $R_H = 4.0$, $c = 0.05$, $F = 2.5$ If the $n = 0.20$, $e = 0.01895$; if $n = 0.30$, $e = 0.0398$.

The intuition of this observation would be the increasing length of the project provides incentive for the contractor to exert effort. Higher- effort means a larger chance to get higher revenue in a longer period as the reimbursement of the costs.

Observation 2

As the operational cost increases, the second best effort is increasing. Then values are set to be: $R_L = 1.5$ and $R_H = 4.0$, $n = 0.20$, $F = 2.5$ If the $c = 0.05$, $e = 0.01895$; if $c = 0.10$, $e = 0.02292$.

If the operational cost is increasing, that means less pure gain in the operation stage, then we should expect the contractor exerts less effort, however, in order to offer incentive to for the contractor, the government would grant longer time for the contractor to operate.



Observation 3

As the construction cost increases, the second best effort increases. Then values are set to be: $R_L = 1.5$ and $R_H = 4.0$, $n = 0.20$, $c = 0.05$ If the $F = 2.5$, $e = 0.01895$; if $F = 1.5$, $e = 0.01749$

Because there is no other monetary transfer, therefore, the government has to use the authorized phase to reimbursement the contractor's construction cost and operation cost. Therefore, the contractor should expect if the construction cost is higher, the government would grant longer operation phase, therefore more incentive to exert more effort.

Observation 4

As the higher revenue outcome R_H decreases, the second best effort decreases. Then values are set to be: $R_L = 1.5$ and $F = 2.5$, $n = 0.20$, $c = 0.05$ If the $R_H = 4.0$, $e = 0.01895$; if $R_H = 3.0$, $e = 0.0072598$ and if $R_H = 2.0$, $e = 0.0009082$.

R_H represents the highest possible gain for the contractor during the operation phase, if such gain decreases, the lure of the project decreases, therefore less incentive to build a good project.

Observation 5

As the lower revenue outcome R_L increases, the second best effort decreases. Then values are set to be: $R_H = 4.0$ and $F = 2.5$, $n = 0.2$, $c = 0.05$ If the $R_L = 1.5$, $e = 0.01895$; if $R_L = 2.5$, $e = 0.006293$.

The graphics show the example when $F = 2.5$,$n = 0.2$,$R_H \in [1.5, 4]$,$e \in [0.001, 1]$,$c \in [0.05, 0.1]$, the left graph shows $R_L = 1.5$ and the right graph shows $R_L = 2.5$

The shaded rigion shows where

$$q'(e)[n(R_H - R_L) + \frac{q''(e)q(e)}{q'(e)^3} - \frac{\frac{1}{q'(e)}(R_L - c) - (F + e^* - \frac{q(e^*)}{q'(e)})(R_H - R_L)}{(R_H - c)(R_L - c)}c] < 1$$

which is the equation that determines the second best effort the government could implement.

Because there two different reimbursement schedules for two different observed outcomes. If difference of two outcomes is small, there is no need for the contractor to exert high effort to distinguish itself out. Therefore as we perceived here, higher R_L results in lower effort.

Conclusion

Based on the value setting we have above, we can observe that the effect of the project length and the operational cost, revenue difference ($R_H - R_L$) and the construction cost have significant effect on the second best effort.

We can also conclude that if we keep the revenue difference the same, decreasing the higher revenue outcome has lower effect on the second best effort compared to an increasing of the lower revenue outcome and both will result in a reduction in such effort.

The contractor's individual rational constraint is ex ante, therefore when the contract is realized, the ex post cost may be not bearable.

There are only two outcomes can be observed in this one period model, the pattern of government would reimburse the contractor would appear like: if the government observed the low outcome, the contractor would be punished; when the outcome is high, the contractor would be rewarded. Therefore ex ante, the contractor would be more likely to exert a proper effort.

APPENDIX A

APPENDIX

A.1 Proof of Proposition 1 of Chapter 1

The operation regulator's problem is:

$$\max_{(\mathbf{t},\mathbf{O},\mathbf{Q})} \sum_{j\in\{H,L\}} \alpha^j[Q^j(n-t^j)s+(1-Q^j)n\cdot s-O^j]$$

$$s.t.$$

$$\forall j,k \in \{H,L\}$$

$$Q^j[t^j(R-c^j)+O^j] \geq Q^k[t^k(R-c^j)+O^k] \tag{IC}$$

$$Q^j[t^j(R-c^j)+O^j] \geq 0 \tag{IR}$$

$$0 \leq t^j \leq n$$

$$0 \leq Q^j \leq 1$$

First I show the objective function is concave(linear) in all the choice variables, and the constrained set is convex. Then I present the lagrangian for the operation regulator. Since the proof bases on analyzing the multipliers and variables' interaction, to avoid confusion, I provide the idea of proof right after presenting the first order conditions and complementary slackness conditions.

I use two different ways to prove that the objective is concave (linear): one is principal minors and the eigenvalue of the Heassian Matrix.

The objective function is concave is all choice variables. The determinant of the Hessian is:

$$D^2 f = \begin{vmatrix} 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & d \\ e & 0 & f & 0 & 0 & 0 \\ 0 & g & 0 & h & 0 & 0 \end{vmatrix}.$$

A matrix is negative definite iff all its kth order leading principal minors alternate in sign, starting from negative and for semi-definiteness, we replace the strict inequalities with weak inequalities.

$$a = -\alpha^H s + \lambda_2(R - c^H)$$

$$b = -\alpha^L s + \lambda_1(R - c^L) - \lambda_2(R - c^H)$$

$$c = -\alpha^H + \lambda_2$$

$$d = -\alpha^L + (\lambda_1 - \lambda_2)$$

$$e = -\alpha^H s + \lambda_2(R - c^H)$$

$$f = \lambda_2 - \alpha^H$$

$$g = -\alpha^L s + \lambda_1(R - c^L) - \lambda_2(R - c^H)$$

$$h = -\alpha^L + \lambda_1 - \lambda_2$$

$$|0| = 0$$

$$2^{nd} m = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}.$$

$$3^{rd}m = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

$$4^{th}m = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}.$$

$$5^{th}m = \begin{vmatrix} 0 & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & 0 & 0 \\ e & 0 & f & 0 & 0 \end{vmatrix}.$$

$$Det2^{nd} = Det3^{nd} = Det4^{th} = Det5^{th} = DetD^2 f = 0$$

Therefore the objective fn is a concave function.

A matrix is negative definite iff all its kth order leading principal minors alternate in sign, starting from negative and for semi-definiteness, we replace the strict inequalities with weak inequalities.

Alternatively, we could also prove the objective function is concave (linear) by changing of variables. I find the eigenvlues after the change of variables.

Redefine $Q^j * t^j = t^j$, where $j = H, L$

The operation agency's objective function can be rewritten as:

$$\alpha^H[(n - t^H)s - O^H] + \alpha^L[(n - t^L)s - O^L]$$

using mathematica to find the eigenvalues for the Hessian matrix , they are $[0, 0, 0, 0]$.

Therefore the objective function of the operation agency is concave (linear).

The constraint set is convex, the Hessian of the constraint set is:

$$D^2 g = \begin{vmatrix} 0 & 0 & 0 & 0 & R-c^H & 0 \\ 0 & 0 & 0 & 0 & 0 & -(R-c^H) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ R-c^H & 0 & 1 & 0 & 0 & 0 \\ 0 & -(R-c^H) & 0 & -1 & 0 & 0 \end{vmatrix}.$$

$$|0| = 0$$

$$2^{nd} m = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}.$$

$$3^{rd} m = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.$$

$$4^{th} m = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}.$$

$$5^{th} m = \begin{vmatrix} 0 & 0 & 0 & 0 & R-c^H \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ R-c^H & 0 & 1 & 0 & 0 \end{vmatrix}.$$

$$Det2^{nd} = Det3^{nd} = Det4^{th} = Det5^{th} = DetD^2g = 0$$

Therefre the constraint set is convex.

For the operation sector, the langrangian is:

$$\begin{aligned}
L = & \alpha^H\{n \cdot s - Q^H[t^H s + O^H]\} + \alpha^L\{n \cdot s - Q^L[t^L s + O^L]\} \\
& + \lambda_1 Q^L[t^L(R - c^L) + O^L] \\
& + \lambda_2\{Q^H[t^H(R - c^H) + O^H] - Q^L[t^L(R - c^H) + O^L]\} \\
& + \lambda_3 t^H + \lambda_4 t^L \\
& + \lambda_5(n - t^H) + \lambda_6(n - t^L) \\
& + \lambda_7 Q^H + \lambda_8 Q^L \\
& + \lambda_9(1 - Q^H) + \lambda_{10}(1 - Q^L)
\end{aligned}$$

F.O.Cs:

t^j:

$$\begin{aligned}
& -\alpha^H Q^H s + \lambda_2 Q^H(R - c^H) + \lambda_3 - \lambda_5 = 0 \\
& -\alpha^L Q^L s + \lambda_1 Q^L(R - c^L) - \lambda_2 Q^L(R - c^H) + \lambda_4 - \lambda_6 = 0
\end{aligned}$$

O^j:

$$\begin{aligned}
& -\alpha^H Q^H + \lambda_2 Q^H = 0 \\
& -\alpha^L Q^L + \lambda_1 Q^L - \lambda_2 Q^L = 0
\end{aligned}$$

Q^j:

$$-\alpha^H(t^H s + O^H) + \lambda_2[t^H(R - c^H) + O^H] + \lambda_7 - \lambda_9 = 0$$
$$-\alpha^L(t^L s + O^L) + \lambda_1[t^L(R - c^L) + O^L] - \lambda_2[t^L(R - c^H) + O^L] + \lambda_8 - \lambda_{10} = 0$$

After organization:

$$Q^H[\lambda_2(R - c^H) - \alpha^H s] + \lambda_3 - \lambda_5 = 0$$
$$Q^L[\lambda_1(R - c^L) - \lambda_2(R - c^H) - \alpha^L s] + \lambda_4 - \lambda_6 = 0$$
$$[\lambda_2 - \alpha^H]Q^H = 0$$
$$[\lambda_1 - \lambda_2 - \alpha^L]Q^L = 0$$
$$-\alpha^H(t^H s + O^H) + \lambda_2[t^H(R - c^H) + O^H] + \lambda_7 - \lambda_9 = 0$$
$$-\alpha^L(t^L s + O^L) + \lambda_1[t^L(R - c^L) + O^L] - \lambda_2[t^L(R - c^H) + O^L] + \lambda_8 - \lambda_{10} = 0$$
$$(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0$$
$$(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0$$

The complementary slackness:

$$\lambda_1 Q^L[t^L(R-c^L)+O^L]=0$$
$$\lambda_2\{Q^H[t^H(R-c^H)+O^H]-Q^L[t^L(R-c^H)+O^L]\}=0$$
$$\lambda_3 t^H=0$$
$$\lambda_4 t^L=0$$
$$\lambda_5(n-t^H)=0$$
$$\lambda_6(n-t^L)=0$$
$$\lambda_7 Q^H=0$$
$$\lambda_8 Q^L=0$$
$$\lambda_9(1-Q^H)=0$$
$$\lambda_{10}(1-Q^L)=0$$

Idea of the proof: To help readers keep track of the proof. I first lay out the objecitves of the proofs. Recall, the government is optimally choose contract (t^j, O^j) with a random probability Q^j.

The authorized phase t^j is the allocation variable for the operation regulator; O^j is the monetary transfer at the operation stage; Q^j is a random device. Therefore for the allocation space, there are in total nine categories available for the operation regulator, which is indicating by $\{t^H=0, 0<t^H<n, t^H=n\}\times\{t^L=0, 0<t^L<n, t^L=n\}$. I prove out of nine categories, there are only three categories will be considered. The rest categories are not optimal contracts candidates.

Lemma 1 and Lemma 2 exclude five categories involving interim solutions of the allocation (t^H, t^L). And we can conclude here, Any non-monotone arrangement for the allocation (t^H, t^L) is not rational, because the high operational efficiency type would always be able to mimic the low efficiency type.

Lemma 6. *$0<t^H<n$ will not be a category in the optimal contract.*

Proof. Based the organizied f.o.c.s, we have

$$\lambda_2 Q^H t^H (R - c^H - s) = 0$$

With $0 < t^H < 1$, there are two possible conditions.

Either $Q^H = 0$ or $\lambda_2 = 0$

If $Q^H = 0$, for optimal contract $(t^{H*}, t^{L*}, O^{H*}, O^{L*})$ where $t^{H*} \in (0,1)$, the government picks it with probabllity zero.

If $\lambda_2 = 0$, according $\lambda_2 Q^H = \alpha^H Q^H$, therefore either $Q^H = 0$,then we arrive at the above conclusion. Or $\lambda_2 = \alpha^H = 0$, therefore according to $\lambda_2 Q^H O^H - \lambda_9 - \lambda_5 n = 0$, since $\lambda_5 = 0$ because this is the multiplier for complementary slackness condition for $t_H = n$, therefore $\lambda_9 = 0$ and it is the multiplier for complementary slackness condition for $Q^H = 1$, then $Q^H \leq 1$.

Since $\alpha^H = 0$ means the updated beilefs the operation regultor has for meeting a high operational efficient type contractor being zero, therefore choosing to let such type operation for a positive length of the project with positive probability contradicts such belief.

□

Lemma 7. *$0 < t^L < n$ will not be a category in the the optimal contract.*

Proof. Suppose $0 < t^L < n$, $t_H = n$

Therefore based on the assumption, $\lambda_5 \geq 0$ and $\lambda_6 = \lambda_4 = 0$,

$$(\lambda_5 - \lambda_3) t^H - \lambda_9 = 0$$

$$(\lambda_6 - \lambda_4) t^L - \lambda_{10} = 0$$

If $\lambda_{10} = 0$, then $t_L = 0$, a contradiction with the assumption.

If $\lambda_{10} > 0$, therefore $Q^L = 1$ therefore $(\lambda_6 - \lambda_4) t^L - \lambda_{10} = 0$ there is no t^L can satisfy the above function.

Under the assumption that $0 < t^L < n$, $t^H = 0$ would not be sensible, because the high efficiency type would always be capable to mimic the low efficiency type.

$\square$

Therefore based on Lemma 2 and Lemma 3, we limited the optimal allocations of the three categories$t^H = n, t^L = n$,$t^H = n, t^L = 0$,$t^H = 0, t^L = 0$. Therefore given these allocation, how the operation regulator uses the random device Q^j to improve utility if any.

Lemma 8. *$(t^H = n, t^L = n)$ is one of the operation regulator's optimal allocation with no randomization.*

Proof. First, $Q^H = 1, 0 < Q^L < 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_{10} = 0$, $\lambda_9 \geq 0$

Since $Q^H = 1$, $\lambda_2 = \alpha^H$ and $\lambda_1 = \alpha_H + \alpha_L$

and $-\alpha^L(n \cdot s + Q^H) - \alpha^L[n(R - c^L) + O^L] - \lambda_2 n(c^L - c^H) - \lambda_{10} = 0$

$$\lambda_{10} = \alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n \neq 0$$

Therefore a contradiction.

Second, $0 < Q^H < 1$ and $Q^L = 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = 0$, $\lambda_{10} \geq 0$

Since $Q^L = 1$, $\lambda_2 = \alpha^H$ and $\lambda_1 = \alpha_H + \alpha_L$

and $-\alpha^H[n \cdot s + O^H] + \alpha^H[n(R - c^H) + O^H] - \lambda_9 = 0$

Therefore $\lambda_9 = \alpha^H n(R - c^H - s) \neq 0$, a contradiction.

Third, $0 < Q^H < 1$ and $0 < Q^L < 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = 0$, $\lambda_1 0 = 0$

$\alpha^H = \alpha^L = 0$ which contradicts the assumption that the operation regulator will grant positive length of the operation phase to the contractor.

$\square$

Lemma 9. *If the allocation $(t^H = n, t^L = n)$ is chosen, that means each type for sure will get such contract with $Q^H = Q^L = 1$.*

Proof. We know that $(t^H = n, t^L = n)$ is chosen with $Q^H = Q^L = 1$ and we find out the condition that such is the optimal allocation.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_{10} \geq 0$, $\lambda_9 \geq 0$

$$
\begin{aligned}
&(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0 \\
&(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0 \\
&\lambda_1 = \alpha^H + \alpha^L \\
&\lambda_2 = \alpha^H \\
&n(R - c^L) + O^L = 0 \\
&n(R - c^H) + O^H - (n(R - c^H) + O^L) = 0 \\
-&\alpha^H n \cdot (s + O^H) + \lambda_2[n(R - c^H) + O^H] - \lambda_9 = 0 \\
-&\alpha^L n \cdot (s + O^L) + \lambda_1[n(R - c^L) + O^L] - \lambda_2[n(R - c^H) + O^L] - \lambda_{10} = 0
\end{aligned}
$$

Therefore, we have:

$$
\begin{aligned}
&(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0 \\
&(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0 \\
&\lambda_9 = \lambda_5 n = \alpha^H n(R - c^H - s) \\
&\lambda_{10} = \alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n \neq 0
\end{aligned}
$$

In order for the solution to be legitimate, $\lambda_{10} > 0$ and $\lambda_9 > 0$ at the same time, and that requires $R - c^H - s > 0$ and $\alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n > 0$ at the same time and that becomes $\alpha^L(R - c^L - s) - \alpha^H(c^L - c^H) > 0$

A.2 Proof of Proposition 2 of Chapter 1

The construction regulator's program:

$$\max_{(\mathbf{P},\mathbf{T})} \sum_{i\in\{H,L\}} \alpha_{ji}[-T_i + Q^j(n-t^j)s + (1-Q^j)n\cdot s - O^j]P_i^j$$

$$s.t.$$

$$\forall i,j,m,k \in \{H,L\}$$

$$[t^{j*}(R-c^j) + O^{j*} + T_i^j - F_i]P_i \geq [t^{k*}(R-c^j) + O^{k*} - T_m^k - F_i]P_m^k \quad \text{(IC)}$$

$$P_i^j[t^j(R-c^j) + O^j + T_i^j - F_i] \geq 0 \quad \text{(IR)}$$

$$0 \leq P_i^j \leq 1$$

First we show the construction regulator's program is concave (linear):

if we redefine $T_i^j P_i^j = T_i^j$, then through using Mathematica, we can compute the eigenvalues of its Hessian Matrix. They are $[0,0,0,0,0,0,0,0]$. Therefore the objective function is concave (linear).

Then we can use the lagrangian.

For the proof of proposition 2, we define: $d^H = R - c^H$ and $d^L = R - c^L$.

For the construction, the utility function of the government is:

$$\begin{aligned}
&\alpha_{HH}P_H^H[(n\cdot s - T_H^H) - \alpha^H(t^H s + T^H)] \\
&+ \alpha_{LH}P_H^L[(n\cdot s - T_H^L) - \alpha^L(t^L s + T^L)] \\
&+ \alpha_{HL}P_L^H[(n\cdot s - T_L^H) - \alpha^H(t^H s + T^H)] \\
&+ \alpha_{LL}P_L^L[(n\cdot s - T_L^L) - \alpha^L(t^L s + T^L)]
\end{aligned}$$

where

$$\alpha^H = \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

$$\alpha^L = \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

And we can rewrite the government's utility function as:

$$\begin{aligned}
&\alpha_{HH} P_H^H [(ns - T_H^H) - \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}} (t^H s + T^H)] \\
&+ \alpha_{LH} P_H^L [(ns - T_H^L) - \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}} (t^L s + T^L)] \\
&+ \alpha_{HL} P_L^H [(ns - T_L^H) - \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}} (t^H s + T^H)] \\
&+ \alpha_{LL} P_L^L [(ns - T_L^L) - \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}} (t^L s + T^L)]
\end{aligned}$$

A type with high construction efficiency can lie in either direction or both, with consideration of only downward IC and transverse incentive constraints.

$$[t^H d^H + T^H + T_H^H - F_H] P_H^H \geq [t^H d^H + T^H + T_L - F_H] P_L^H$$

$$[t^H d^H + T^H + T_H^H - F_H] P_H^H \geq [t^L d^H + T^L + T_H - F_H] P_H^L$$

$$[t^H d^H + T^H + T_H^H - F_H] P_H^H \geq [t^L d^H + T^L + T_L - F_H] P_L^L$$

$$[t^L d^L + T^L + T_H^L - F_H] P_H^L \geq [t^L d^L + T^L + T_L^L - F_H] P_L^L$$

$$[t^L d^L + T^L + T_H^L - F_H] P_H^L \geq [t^H d^L + T^H + T_L^H - F_H] P_L^H$$

$$[t^H d^H + T^H + T_L^H - F_L] P_L^H \geq [t^L d^H + T^L + T_L^L - F_L] P_L^L$$

$$[t^H d^H + T^H + T_L^H - F_L] P_L^H \geq [t^L d^H + T^L + T_H^L - F_L] P_H^L$$

From IR we know that:

$$[t^L d^L + T^L + T^L_L - F_L]P^L_L = 0$$

So the lagrangian for the constructions problem is:

L= $\alpha_{HH}P^H_H[(ns - T^H_H) - \frac{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}}{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}+P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}(t^H s + T^H)]$
$+ \alpha_{LH}P^L_H[(ns - T^L_H) - \frac{P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}+P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}(t^L s + T^L)]$
$+ \alpha_{HL}P^H_L[(ns - T^H_L) - \frac{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}}{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}+P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}(t^H s + T^H)]$
$+ \alpha_{LL}P^L_L[(ns - T^L_L) - \frac{P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}{P^H_H\alpha_{HH}+P^H_L\alpha_{HL}+P^L_H\alpha_{LH}+P^L_L\alpha_{LL}}(t^L s + T^L)]$
$+ \lambda_1[t^{H*}d^H + T^{H*} + T^H_H - F_H]P^H_H$
$+ \lambda_2[t^{H*}d^H + T^{H*} + T^H_L - F_L]P^H_L$
$+ \lambda_3[t^{L*}d^L + T^{L*} + T^L_H - F_H]P^L_H$
$+ \lambda_4[t^{L*}d^L + T^{L*} + T^L_L - F_L]P^L_L$
$+ \lambda_5[t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^H d^H + T^H + T^H_L - F_H]P^H_L$
$+ \lambda_6[t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^L d^H + T^L + T^L_H - F_H]P^L_H$
$+ \lambda_7[t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^L d^H + T^L + T^H_L - F_H]P^L_L$
$+ \lambda_8[t^L d^L + T^L + T^L_H - F_H]P^L_H - [t^L d^L + T^L + T_L - F_H]P^L_L$
$+ \lambda_9[t^L d^L + T^L + T^L_H - F_H]P^L_H - [t^H d^L + T^H + T^H_L - F_H]P^H_L$
$+ \lambda_{10}[t^H d^H + T^H + T^H_L - F_L]P^H_L - [t^L d^H + T^L + T^L_L - F_L]P^L_L$
$+ \lambda_{11}[t^H d^H + T^H + T^H_L - F_L]P^H_L - [t^L d^H + T^L + T^L_H - F_L]P^L_H$
$+ \lambda_{12}P^H_H + \lambda_{13}P^L_H + \lambda_{14}P^H_L + \lambda_{15}P^L_L$
$+ \lambda_{16}(1 - P^H_H) + \lambda_{17}(1 - P^L_H) + \lambda_{18}(1 - P^H_L) + \lambda_{19}(1 - P^L_L)$

The first order conditions are as following:

$P_{ij}s$

$$\alpha_{HH}[(ns-T_H^H)-\alpha^H(t^Hs+T^H)]+(\lambda_1+\lambda_5+\lambda_6+\lambda_7)(t^Hd^H+T^H+T_H^H-F_H)+\lambda_{12}-\lambda_{16}=0$$

$$\alpha_{HL}[(ns-T_L^H)-\alpha^H(t^Hs+T^H)]+(\lambda_2-\lambda_5+\lambda_{10}+\lambda_{11}-\lambda_9)(t^Hd^H+T^H+T_L^H-F_L)$$

$$-\lambda_5\Delta F-\lambda_9(t^H\Delta c-\Delta F)+\lambda_{13}-\lambda_{17}=0$$

$$\alpha_{LH}[(ns-T_H^L)-\alpha^L(t^Ls+T^L)]+(\lambda_3-\lambda_6+\lambda_8+\lambda_9-\lambda_{11})(t^Ld^L+T^L+T_H^L-F_H)$$

$$-\lambda_6(t^L\Delta c)-\lambda_{11}(\Delta F-t^L\Delta c)+\lambda_{14}-\lambda_{18}=0$$

$$\alpha_{LL}[(ns-T_L^L)-\alpha^L(t^Ls+T^L)]+(\lambda_4-\lambda_7-\lambda_8-\lambda_{10})(t^Ld^L+T^L+T_L^L-F_L)$$

$$-(\lambda_7+\lambda_{10})t^L\Delta c-(\lambda_7+\lambda_8)\Delta F+\lambda_{15}-\lambda_{19}=0$$

T_i^js

$$P_H^H[\alpha_{HH}-(\lambda_1+\lambda_5+\lambda_6+\lambda_7)]=0$$
$$P_L^H[\alpha_{HL}+(\lambda_2-\lambda_5+\lambda_{10}+\lambda_{11}-\lambda_9)]=0$$
$$P_H^L[\alpha_{LH}+(\lambda_3-\lambda_6+\lambda_8+\lambda_9-\lambda_{11})]=0$$
$$P_L^L[\alpha_{LL}+(\lambda_4-\lambda_7-\lambda_8-\lambda_{10})]=0$$

Proof. When $R - s > c_L + \frac{\alpha^H \Delta c}{\alpha^L}$ and $n\Delta c > \Delta F$:

$$\lambda_1 = 0$$

$$\lambda_2 = \alpha_H$$

$$\lambda_3 = \alpha_L$$

$$\lambda_4 = 1$$

$$\lambda_5 = \lambda_6 = 0$$

$$\lambda_7 = \alpha_H$$

$$\lambda_8 = 0$$

$$\lambda_9 = 0$$

$$\lambda_{10} = 0$$

$$\lambda_{11} = 0$$

$$\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = 0$$

$$\lambda_{16} = \alpha_H[ns + (d^H - s)t^{H*} - F_H]$$

$$\lambda_{17} = \alpha_H[ns + (d^H - s)t^{H*} - F_L]$$

$$\lambda_{18} = \alpha_H[ns + (d^L - s)t^{L*} - F_H]$$

$$\lambda_{19} = 0$$

$$\lambda_{20} = \lambda_{21} = 0$$

$$\lambda_{22} = \alpha_H(nd^L + nd^H - 2F_H)$$

$$\lambda_{23} = \alpha_L(nd^H - F_L)$$

$$P_H^H = P_H^L = P_L^H = 1$$

$$P_L^L = 0$$

$$T_L^H = F_H$$

$$T_H^j = F_H$$

And the solution is consistant with the belief of the operation sector as:

If α_{LL} is small and:

$$\alpha^H = \frac{\alpha_{HH} + \alpha_{HL}}{\alpha_{HH} + \alpha_{HL} + \alpha_{LH}}$$

$$\alpha^L = \frac{\alpha_{LH}}{\alpha_{HH} + \alpha_{HL} + \alpha_{LH}}$$

If α_L is big, full pooling for every type.

By observing the first order conditions of $Q^j s$ above:

$$\alpha^H Q^H ns + \lambda_5 n = \lambda_9$$

$$\alpha^L Q^L ns + \lambda_6 n = \lambda_{10}$$

Assume that the operation sector will grant the project to the contractor with a positive probability $0 < Q^H < 1$,

Therefore according to the complementary slackness conditions:

$$\lambda^9 = 0$$

$$\lambda_5 \geq 0$$

The above equation stands only when $Q^H = 0$, which is a contradiction. The same logic can be used to show Q^L.

First Order conditions:

$$\alpha_H[P_H(ns - F_H)] + \lambda_1 - \lambda_3 = 0$$
$$P_L[\alpha_L(ns - F_L - \alpha_H \Delta F] + \lambda_2 - \lambda_4 = 0$$

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_5 = \lambda_6 = \lambda_9 = \lambda_{11} = 0$$
$$\lambda_4 = 3\alpha_L + \alpha_H$$
$$\lambda_7 = \alpha_H$$
$$\lambda_8 = \alpha_L$$
$$\lambda_{10} = \alpha_L$$
$$\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = 0$$
$$\lambda_{16} = \alpha_H[ns + (d_H - s)t^{H*} - F_H]$$
$$\lambda_{17} = \alpha_L[ns + (d_H - s)t^{H*} - F_L]$$
$$\lambda_{18} = \alpha_H[ns + (d_L - s)t^{L*} - F_H]$$
$$\lambda_{19} = \lambda_{20} = \lambda_{21} = \lambda_{23} = 0$$
$$\lambda_{22} = \alpha_H \Delta c$$

$$P_H^H = P_H^L = P_L^H = 1$$

$$P_L^L = 0$$

$$T_L = F_L - n\Delta c$$

$$T_H = F_H$$

For Proposition 3, we can also prove that the construction agent's objective function is actually a concave (linear) function with eigenvalues of its Hessian Matrix being $[0, 0, 0, 0]$. Then in this case, it is a basic one dimensional screening problem. Therefore we following the "standard" approach to find the 'optimality', with the IR constraint binding for the low efficiency type and the IC binding for the high efficiency type. Once I solve the problem, the monotonicity of the solution is checked.

$\square$

A.3 Proof in Section 3 of Chapter 1

In this section, I claim:

The revelation principle cannot be applied properly with information restrictions potentially implied by the Chinese Wall.

I construct an untruthful equilibrium such that no truthful equilibrium can generate weakly better utility for the government by using the revelation principle. In such case, by implementing the menu that generates the equivalent outcome as the optimal contract with the full report (or like the untruthful equilibrium), the government agencies can improve utilities.

The menu is: the construction agency pays construction F_H and the operation agency charges franchise fee $n * (R - c^l)$ and let the contractor operate for n years. This menu is optimal when the following conditions are all true: the contractor is much more operational efficient compared to the government; the information rent paid on the operation stage is higher than that of the construction stage and the contractor is unlikely to be the low efficiency type on both dimensions. Therefore by paying every type as if they were high efficiency on the construction dimension, the construction agency can eliminate the rent paying. Because the information rent is higher on the operation dimension. The type with high operational efficiency but low construction efficiency still participates because it is still profitable given the assumption the outside option is zero.

First we prove the above menu generates an untruthful equilibrium in the game with Chinese Wall's information restriction. Given the above menu, every type besides the high operation efficiency and low construction efficiency type (c^H, F_L) is indifferent to tell the truth. (c^H, F_L) lies about construction cost and suffers a lost at the construction stage $(F_L - F_H)$, but this loss can be compensated by its information rent earned at the operation stage $n(c^L - c^H)$, given that the information rent paid on the operation dimension is higher than that on the construction. Therefore every type is best responding to the announced menu.

For the government, knowing that all types but the type (c^H, F_L) tell the truth, the government announces the above menu which excludes the type with low efficiency on both dimensions. The government gains by not paying the information rent on the construction dimension. Since the contractor is unlikely to the type with low efficiency on both dimension, the lost of contribution from the type with low efficiency on both dimensions is smaller than the saving on the information rent paying on the construction dimension. The government is also best responding to the contractor's untruthful report. Therefore I prove this is an untruthful equilibrium.

According to Proposition 1, 2, and 3, if we use the revelation principle under informational restriction of the Chinese Wall, the construction agency is not able to distinguish type (c^H, F_L) and type (c^L, F_L), the construction contract is alway a pooling contract for both of them, therefore an additional information rent has to be paid while it may be avoidable. Hence I prove the claim that the revelation principle cannot be applied properly under the Chinese Wall informational restriction.

A.4 Proof of Two-Sector of Chapter 2

The operation regulator's problem is:

$$\max_{(\mathbf{t},\mathbf{O},\mathbf{Q})} \sum_{j\in\{H,L\}} \alpha^j[Q^j(n-t^j)s+(1-Q^j)n\cdot s-O^j]$$

$$s.t.$$

$$\forall j,k \in \{H,L\}$$

$$Q^j[t^j(R-c^j)+O^j] \geq Q^k[t^k(R-c^j)+O^k] \tag{IC}$$

$$Q^j[t^j(R-c^j)+O^j] \geq 0 \tag{IR}$$

$$0 \leq t^j \leq n$$

$$0 \leq Q^j \leq 1$$

First I show the objective function is concave in all the choice variables, and the constrained set is convex. Then I present the lagrangian for the operation regulator. Since the proof bases on analyzing the multipliers and variables' interaction, to avoid confusion, I provide the idea of proof right after presenting the first order conditions and complementary slackness conditions.

The objective function is concave is all choice variables. The determinant of the Hessian is:

$$D^2 f = \begin{vmatrix} 0 & 0 & 0 & 0 & a & 0 \\ 0 & 0 & 0 & 0 & 0 & b \\ 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & d \\ e & 0 & f & 0 & 0 & 0 \\ 0 & g & 0 & h & 0 & 0 \end{vmatrix}.$$

$$a = -\alpha^H s + \lambda_2(R - c^H)$$

$$b = -\alpha^L s + \lambda_1(R - c^L) - \lambda_2(R - c^H)$$

$$c = -\alpha^H + \lambda_2$$

$$d = -\alpha^L + (\lambda_1 - \lambda_2)$$

$$e = -\alpha^H s + \lambda_2(R - c^H)$$

$$f = \lambda_2 - \alpha^H$$

$$g = -\alpha^L s + \lambda_1(R - c^L) - \lambda_2(R - c^H)$$

$$h = -\alpha^L + \lambda_1 - \lambda_2$$

$$Det D^2 f = 0$$

Therefore the objective fn is a concave function.

The constraint set is convex, the Hessian of the constraint set is:

$$D^2 g = \begin{vmatrix} 0 & 0 & 0 & 0 & R - c^H & 0 \\ 0 & 0 & 0 & 0 & 0 & -(R - c^H) \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ R - c^H & 0 & 1 & 0 & 0 & 0 \\ 0 & -(R - c^H) & 0 & -1 & 0 & 0 \end{vmatrix}.$$

$$Det D^2 g = 0$$

Therefre the constraint set is convex.

For the operation sector, the langrangian is:

$$
\begin{aligned}
L =& \alpha^H\{n \cdot s - Q^H[t^H s + O^H]\} + \alpha^L\{n \cdot s - Q^L[t^L s + O^L]\} \\
& + \lambda_1 Q^L[t^L(R - c^L) + O^L] \\
& + \lambda_2\{Q^H[t^H(R - c^H) + O^H] - Q^L[t^L(R - c^H) + O^L]\} \\
& + \lambda_3 t^H + \lambda_4 t^L \\
& + \lambda_5(n - t^H) + \lambda_6(n - t^L) \\
& + \lambda_7 Q^H + \lambda_8 Q^L \\
& + \lambda_9(1 - Q^H) + \lambda_{10}(1 - Q^L)
\end{aligned}
$$

F.O.Cs:

t^j:

$$
\begin{aligned}
& - \alpha^H Q^H s + \lambda_2 Q^H(R - c^H) + \lambda_3 - \lambda_5 = 0 \\
& - \alpha^L Q^L s + \lambda_1 Q^L(R - c^L) - \lambda_2 Q^L(R - c^H) + \lambda_4 - \lambda_6 = 0
\end{aligned}
$$

O^j:

$$
\begin{aligned}
& - \alpha^H Q^H + \lambda_2 Q^H = 0 \\
& - \alpha^L Q^L + \lambda_1 Q^L - \lambda_2 Q^L = 0
\end{aligned}
$$

Q^j:

$$
\begin{aligned}
& - \alpha^H(t^H s + O^H) + \lambda_2[t^H(R - c^H) + O^H] + \lambda_7 - \lambda_9 = 0 \\
& - \alpha^L(t^L s + O^L) + \lambda_1[t^L(R - c^L) + O^L] - \lambda_2[t^L(R - c^H) + O^L] + \lambda_8 - \lambda_{10} = 0
\end{aligned}
$$

After organization:

$$Q^H[\lambda_2(R-c^H)-\alpha^H s]+\lambda_3-\lambda_5=0$$

$$Q^L[\lambda_1(R-c^L)-\lambda_2(R-c^H)-\alpha^L s]+\lambda_4-\lambda_6=0$$

$$[\lambda_2-\alpha^H]Q^H=0$$

$$[\lambda_1-\lambda_2-\alpha^L]Q^L=0$$

$$-\alpha^H(t^H s+O^H)+\lambda_2[t^H(R-c^H)+O^H]+\lambda_7-\lambda_9=0$$

$$-\alpha^L(t^L s+O^L)+\lambda_1[t^L(R-c^L)+O^L]-\lambda_2[t^L(R-c^H)+O^L]+\lambda_8-\lambda_{10}=0$$

$$(\lambda_5-\lambda_3)t^H-\lambda_9=0$$

$$(\lambda_6-\lambda_4)t^L-\lambda_{10}=0$$

The complementary slackness:

$$\lambda_1 Q^L[t^L(R-c^L)+O^L]=0$$

$$\lambda_2\{Q^H[t^H(R-c^H)+O^H]-Q^L[t^L(R-c^H)+O^L]\}=0$$

$$\lambda_3 t^H=0$$

$$\lambda_4 t^L=0$$

$$\lambda_5(n-t^H)=0$$

$$\lambda_6(n-t^L)=0$$

$$\lambda_7 Q^H=0$$

$$\lambda_8 Q^L=0$$

$$\lambda_9(1-Q^H)=0$$

$$\lambda_{10}(1-Q^L)=0$$

Idea of the proof: To help readers keep track of the proof. I will first lay out the objecitves of the proofs. Recall, the government is optimally choose contract (t^j, O^j) with a random probability Q^j.

The authorized phase t^j is the allocation variable for the operation regulator; O^j is the monetary transfer at the operation stage; Q^j is a random device. Therefore for the allocation space, there are in total nine categories available for the operation regulator, which is indicating by $\{t^H = 0, 0 < t^H < n, t^H = n\} \times \{t^L = 0, 0 < t^L < n, t^L = n\}$. I will first prove out of nine categories, there are only three categories will be considered. The rest categories are not optimal contracts candidates.

Lemma 1 and Lemma 2 exclude five categories involving interim solutions of the allocation (t^H, t^L). And we can conclude here, Any non-monotone arrangement for the allocation (t^H, t^L) is not rational, because the high operational efficiency type would always be able to mimc the low efficiency type.

Lemma 10. *$0 < t^H < n$ will not be a category in the optimal contract.*

Proof. Based the organzied focs, we have

$$\lambda_2 Q^H t^H (R - c^H - s) = 0$$

With $0 < t^H < 1$, there are two possible conditions.

Either $Q^H = 0$ or $\lambda_2 = 0$

If $Q^H = 0$, for optimal contract $(t^{H*}, t^{L*}, O^{H*}, O^{L*})$ where $t^{H*} \in (0, 1)$, the government will pick with probabllity zero.

If $\lambda_2 = 0$, according $\lambda_2 Q^H = \alpha^H Q^H$, therefore either $Q^H = 0$,then we will get the above conclusion. Or $\lambda_2 = \alpha^H = 0$, therefore according to $\lambda_2 Q^H O^H - \lambda_9 - \lambda_5 n = 0$, since $\lambda_5 = 0$ because this is the multiplier for complementary slackness condition for $t_H = n$, therefore $\lambda_9 = 0$ and it is the multiplier for complementary slackness condition for $Q^H = 1$, then $Q^H \leq 1$.

SInce $\alpha^H = 0$ means the updated beilefs the operation regultor has for meeting a high operational efficient type contractor will be zero, therefore choosing to let such type operation for a positive length of the project with positive probability will contradicts such belief.

□

Lemma 11. $0 < t^L < n$ *will not be a category in the the optimal contract.*

Proof. Suppose $0 < t^L < n$, $t_H = n$

Therefore based on the assumption, $\lambda_5 \geq 0$ and $\lambda_6 = \lambda_4 = 0$,

$$(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0$$

$$(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0$$

If $\lambda_{10} = 0$, then $t_L = 0$, a contradiction with the assumption.

If $\lambda_{10} > 0$, therefore $Q^L = 1$ therefore $(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0$ there is no t^L can satisfy the above function.

Under the assumption that $0 < t^L < n$, $t^H = 0$ would not be sensible, because the high efficiency type would always be capable to mimic the low efficiency type.

□

Therefore based on Lemma 2 and Lemma 3, we limited the optimal allocations of the three categories$t^H = n, t^L = n$,$t^H = n, t^L = 0$,$t^H = 0, t^L = 0$. Therefore we will see, given these allocation, how the operation regulator will use the random device Q^j to improve utility if any.

Lemma 12. $(t^H = n, t^L = n)$ *is one of the operation regulator's optimal allocation with no randomization.*

Proof. First, $Q^H = 1, 0 < Q^L < 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_{10} = 0$, $\lambda_9 \geq 0$

Since $Q^H = 1$, $\lambda_2 = \alpha^H$ and $\lambda_1 = \alpha_H + \alpha_L$

and $-\alpha^L(n \cdot s + Q^H) - \alpha^L[n(R - c^L) + O^L] - \lambda_2 n(c^L - c^H) - \lambda_{10} = 0$

$$\lambda_{10} = \alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n \neq 0$$

Therefore a contradiction.

Second, $0 < Q^H < 1$ and $Q^L = 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = 0$, $\lambda_{10} \geq 0$

Since $Q^L = 1$, $\lambda_2 = \alpha^H$ and $\lambda_1 = \alpha_H + \alpha_L$

and $-\alpha^H[n \cdot s + O^H] + \alpha^H[n(R - c^H) + O^H] - \lambda_9 = 0$

Therefore $\lambda_9 = \alpha^H n(R - c^H - s) \neq 0$, a contradiction.

Third, $0 < Q^H < 1$ and $0 < Q^L < 1$ is not viable.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_9 = 0$, $\lambda_1 0 = 0$

$\alpha^H = \alpha^L = 0$ which contradicts the assumption that the operation regulator will grant positive length of the operation phase to the contractor.

□

Lemma 13. *If the allocation $(t^H = n, t^L = n)$ is chosen, that means each type for sure will get such contract with $Q^H = Q^L = 1$.*

Proof. We know that $(t^H = n, t^L = n)$ is chosen with $Q^H = Q^L = 1$ and we will find out the condition that such is the optimal allocation.

Based on the assumptions, $\lambda_7 = 0$, $\lambda_8 = 0$, $\lambda_{10} \geq 0$, $\lambda_9 \geq 0$

$$(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0$$
$$(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0$$
$$\lambda_1 = \alpha^H + \alpha^L$$
$$\lambda_2 = \alpha^H$$
$$n(R - c^L) + O^L = 0$$
$$n(R - c^H) + O^H - (n(R - c^H) + O^L) = 0$$
$$-\alpha^H n \cdot (s + O^H) + \lambda_2[n(R - c^H) + O^H] - \lambda_9 = 0$$
$$-\alpha^L n \cdot (s + O^L) + \lambda_1[n(R - c^L) + O^L] - \lambda_2[n(R - c^H) + O^L] - \lambda_{10} = 0$$

Therefore, we will have:

$$(\lambda_5 - \lambda_3)t^H - \lambda_9 = 0$$

$$(\lambda_6 - \lambda_4)t^L - \lambda_{10} = 0$$

$$\lambda_9 = \lambda_5 n = \alpha^H n(R - c^H - s)$$

$$\lambda_{10} = \alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n \neq 0$$

In order for the solution to be legitimate, $\lambda_{10} > 0$ and $\lambda_9 > 0$ at the same time, and that requires $R - c^H - s > 0$ and $\alpha^L n(R - c^L - s) - \alpha^H(c^L - c^H)n > 0$ at the same time and that becomes $\alpha^L(R - c^L - s) - \alpha^H(c^L - c^H) > 0$

For the construction, the utility function is:

$$U_G = \alpha_{HH} P_H^H[(ns - T_H^H) - \alpha^H(t^H s + T^H)]$$

$$+\alpha_{LH} P_H^L[(ns - T_H^L) - \alpha^L(t^L s + T^L)]$$

$$+\alpha_{HL}P_L^H[(ns - T_L^H) - \alpha^H(t^H s + T^H)]$$

$$+\alpha_{LL}P_L^L[(ns - T_L^L) - \alpha^L(t^L s + T^L)]$$

$$\alpha^H = \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

$$\alpha^L = \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}$$

$$U_G = \alpha_{HH}P_H^H[(ns - T_H^H) - \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}}(t^H s + T^H)]$$

$$+\alpha_{LH}P_H^L[(ns - T_H^L) - \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}(t^L s + T^L)]$$

$$+\alpha_{HL}P_L^H[(ns - T_L^H) - \frac{\sum_i P_i^H \alpha_{Hi}}{\sum_i \sum_j P_i^j \alpha_{ji}}(t^H s + T^H)]$$

$$+\alpha_{LL}P_L^L[(ns - T_L^L) - \frac{\sum_i P_i^L \alpha_{Li}}{\sum_i \sum_j P_i^j \alpha_{ji}}(t^L s + T^L)]$$

A type with high construction efficiency can lie in either direction or both, with consideration of only downward ic and transverse incentive constraints.

$$[t^H d^H + T^H + T_H^H - F_H]P_H^H \geq [t^H d^H + T^H + T_L - F_H]P_L^H$$

$$[t^H d^H + T^H + T_H^H - F_H]P_H^H \geq [t^L d^H + T^L + T_H - F_H]P_H^L$$

$$[t^H d^H + T^H + T_H^H - F_H]P_H^H \geq [t^L d^H + T^L + T_L - F_H]P_L^L$$

$$[t^L d^L + T^L + T_H^L - F_H]P_H^L \geq [t^L d^L + T^L + T_L^L - F_H]P_L^L$$

$$[t^L d^L + T^L + T_H^L - F_H]P_H^L \geq [t^H d^L + T^H + T_L^H - F_H]P_L^H$$

$$[t^H d^H + T^H + T_L^H - F_L]P_L^H \geq [t^L d^H + T^L + T_L^L - F_L]P_L^L$$

$$[t^H d^H + T^H + T^H_L - F_L]P^H_L \geq [t^L d^H + T^L + T^L_H - F_L]P^L_H$$

From IR we know that:

$$[t^L d^L + T^L + T^L_L - F_L]P^L_L = 0$$

So the lagrangian for the constructions problem is:

$$L = \alpha_{HH} P^H_H [(ns - T^H_H) - \frac{P^H_H \alpha_{HH} + P^H_L \alpha_{HL}}{P^H_H \alpha_{HH} + P^H_L \alpha_{HL} + P^L_H \alpha_{LH} + P^L_L \alpha_{LL}} (t^H s + T^H)]$$

$$+\alpha_{LH} P^L_H [(ns - T^L_H) - \frac{P^L_H \alpha_{LH} + P^L_L \alpha_{LL}}{P^H_H \alpha_{HH} + P^H_L \alpha_{HL} + P^L_H \alpha_{LH} + P^L_L \alpha_{LL}} (t^L s + T^L)]$$

$$+\alpha_{HL} P^H_L [(ns - T^H_L) - \frac{P^H_H \alpha_{HH} + P^H_L \alpha_{HL}}{P^H_H \alpha_{HH} + P^H_L \alpha_{HL} + P^L_H \alpha_{LH} + P^L_L \alpha_{LL}} (t^H s + T^H)]$$

$$+\alpha_{LL} P^L_L [(ns - T^L_L) - \frac{P^L_H \alpha_{LH} + P^L_L \alpha_{LL}}{P^H_H \alpha_{HH} + P^H_L \alpha_{HL} + P^L_H \alpha_{LH} + P^L_L \alpha_{LL}} (t^L s + T^L)]$$

$$+\lambda_1 [t^{H*} d^H + T^{H*} + T^H_H - F_H]P^H_H$$

$$+\lambda_2 [t^{H*} d^H + T^{H*} + T^H_L - F_L]P^H_L$$

$$+\lambda_3 [t^{L*} d^L + T^{L*} + T^L_H - F_H]P^L_H$$

$$+\lambda_4 [t^{L*} d^L + T^{L*} + T^L_L - F_L]P^L_L$$

$$+\lambda_5 [t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^H d^H + T^H + T^H_L - F_H]P^H_L$$

$$+\lambda_6 [t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^L d^H + T^L + T^L_H - F_H]P^L_H$$

$$+\lambda_7 [t^H d^H + T^H + T^H_H - F_H]P^H_H - [t^L d^H + T^L + T^H_L - F_H]P^L_L$$

$$+\lambda_8 [t^L d^L + T^L + T^L_H - F_H]P^L_H - [t^L d^L + T^L + T_L - F_H]P^L_L$$

$$+\lambda_9[t^L d^L + T^L + T^L_H - F_H]P^L_H - [t^H d^L + T^H + T^H_L - F_H]P^H_L$$

$$+\lambda_{10}[t^H d^H + T^H + T^H_L - F_L]P^H_L - [t^L d^H + T^L + T^L_L - F_L]P^L_L$$

$$+\lambda_{11}[t^H d^H + T^H + T^H_L - F_L]P^H_L - [t^L d^H + T^L + T^L_H - F_L]P^L_H$$

$$+\lambda_{12}P^H_H + \lambda_{13}P^L_H + \lambda_{14}P^H_L + \lambda_{15}P^L_L$$

$$+\lambda_{16}(1 - P^H_H) + \lambda_{17}(1 - P^L_H) + \lambda_{18}(1 - P^H_L) + \lambda_{19}(1 - P^L_L)$$

The first order conditions are as following:

$P_{ij}s$

$$\alpha_{HH}[(ns - T^H_H) - \alpha^H(t^H s + T^H)] + (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)(t^H d^H + T^H + T^H_H - F_H) + \lambda_{12} - \lambda_{16} = 0$$

$$\alpha_{HL}[(ns - T^H_L) - \alpha^H(t^H s + T^H)] + (\lambda_2 - \lambda_5 + \lambda_{10} + \lambda_{11} - \lambda_9)(t^H d^H + T^H + T^H_L - F_L)$$

$$-\lambda_5 \Delta F - \lambda_9(t^H \Delta c - \Delta F) + \lambda_{13} - \lambda_{17} = 0$$

$$\alpha_{LH}[(ns - T^L_H) - \alpha^L(t^L s + T^L)] + (\lambda_3 - \lambda_6 + \lambda_8 + \lambda_9 - \lambda_{11})(t^L d^L + T^L + T^L_H - F_H)$$

$$-\lambda_6(t^L \Delta c) - \lambda_{11}(\Delta F - t^L \Delta c) + \lambda_{14} - \lambda_{18} = 0$$

$$\alpha_{LL}[(ns - T^L_L) - \alpha^L(t^L s + T^L)] + (\lambda_4 - \lambda_7 - \lambda_8 - \lambda_{10})(t^L d^L + T^L + T^L_L - F_L)$$

$$-(\lambda_7 + \lambda_{10})t^L \Delta c - (\lambda_7 + \lambda_8)\Delta F + \lambda_{15} - \lambda_{19} = 0$$

$T_i^j s$

$$P_H^H[\alpha_{HH} - (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)] = 0$$

$$P_L^H[\alpha_{HL} + (\lambda_2 - \lambda_5 + \lambda_{10} + \lambda_{11} - \lambda_9)] = 0$$

$$P_H^L[\alpha_{LH} + (\lambda_3 - \lambda_6 + \lambda_8 + \lambda_9 - \lambda_{11})] = 0$$

$$P_L^L[\alpha_{LL} + (\lambda_4 - \lambda_7 - \lambda_8 - \lambda_{10})] = 0$$

Proof of Proposition 3

When $R - s > c_L + \frac{\alpha^H \Delta c}{\alpha^L}$ and $n\Delta c > \Delta F$:

$$\lambda_1 = 0$$

$$\lambda_2 = \alpha_H$$

$$\lambda_3 = \alpha_L$$

$$\lambda_4 = 1$$

$$\lambda_5 = \lambda_6 = 0$$

$$\lambda_7 = \alpha_H$$

$$\lambda_8 = 0$$

$$\lambda_9 = 0$$

$$\lambda_{10} = 0$$

If α_L is big.

$$\lambda_{11} = 0$$

$$\lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = 0$$

$$\lambda_{16} = \alpha_H[ns + (d^H - s)t^{H*} - F_H]$$

$$\lambda_{17} = \alpha_{H[ns+(d^H-s)t^{H*}-F_L]}$$

$$\lambda_{18} = \alpha_H[ns + (d^L - s)t^{L*} - F_H]$$

$$\lambda_{19} = 0$$

$$\lambda_{20} = \lambda_{21} = 0$$

$$\lambda_{22} = \alpha_H(nd^L + nd^H - 2F_H)$$

$$\lambda_{23} = \alpha_L(nd^H - F_L)$$

$$P_H^H = P_H^L = P_L^H = 1$$

$$P_L^L = 0$$

$$T_L^H = F_H$$

$$T_H^j = F_H$$

And the solution is consistant with the belief of the operation sector as:

If α_{LL} is very small,

$$\alpha^H = \frac{\alpha_{HH} + \alpha_{HL}}{\alpha_{HH} + \alpha_{HL} + \alpha_{LH}}$$

$$\alpha^L = \frac{\alpha_{LH}}{\alpha_{HH} + \alpha_{HL} + \alpha_{LH}}$$

If α_L is big.

Full pooling for every type.

$\square$

A.5 Proof of Single Regulator Model of Chapter 2

The Lagrangian for the general model:

$$L = \alpha_{HH}\{[n - t_{HH}]s - T_{HH}\}P_{HH} + \alpha_{HL}\{[n - t_{HL}]s - T_{HL}\}P_{HL} +$$

$$\alpha_{LH}\{[n - t_{LH}]s - T_{LH}\}P_{LH} + \alpha_{LL}\{[n - t_{LL}]s - T_{LL}\}P_{LL}$$

$$+\lambda_1\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} + \lambda_2\{t_{HL}d_H + T_{HL} - F_L\}P_{HL}$$

$$+\lambda_3\{t_{LH}d_L + T_{LH} - F_H\}P_{LH} + \lambda_4\{t_{LL}d_L + T_{LL} - F_L\}P_{LL}$$

$$\lambda_5[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{HL}d_H + T_{HL} - F_H\}P_{HL}]$$

$$\lambda_6[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{LH}d_H + T_{LH} - F_H\}P_{LH}]$$

$$\lambda_7[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{LL}d_H + T_{LL} - F_H\}P_{LL}]$$

$$\lambda_8[\{t_{HL}d_H + T_{HL} - F_L\}P_{HL} - \{t_{LL}d_H + T_{LL} - F_L\}P_{LL}]$$

$$\lambda_9[\{t_{LH}d_L + T_{LH} - F_H\}P_{LH} - \{t_{LL}d_L + T_{LL} - F_H\}P_{LL}]$$

$$\lambda_{10}P_{HH} + \lambda_{11}P_{HL} + \lambda_{12}P_{LH} + \lambda_{13}P_{LL}$$

$$\lambda_{14}(1 - P_{HH}) + \lambda_{15}(1 - P_{HL}) + \lambda_{16}(1 - P_{LH}) + \lambda_{17}(1 - P_{LL})$$

$$\lambda_{18}t_{HH} + \lambda_{19}t_{HL} + \lambda_{20}t_{LH} + \lambda_{21}t_{LL}$$

$$\lambda_{22}(n - t_{HH}) + \lambda_{23}(n - t_{HL}) + \lambda_{24}(n - t_{LH}) + \lambda_{25}(n - t_{LL})$$

Complementary Slackness:

$$\lambda_1\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} = 0$$

$$\lambda_2\{t_{HL}d_H + T_{HL} - F_L\}P_{HL} = 0$$

$$\lambda_3\{t_{LH}d_L + T_{LH} - F_H\}P_{LH} = 0$$

$$\lambda_4\{t_{LL}d_L + T_{HH} - F_L\}P_{LL} = 0$$

$$\lambda_5[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{HL}d_H + T_{HL} - F_H\}P_{HL}] = 0$$

$$\lambda_6[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{LH}d_H + T_{LH} - F_H\}P_{LH}] = 0$$

$$\lambda_7[\{t_{HH}d_H + T_{HH} - F_H\}P_{HH} - \{t_{LL}d_H + T_{LL} - F_H\}P_{LL}] = 0$$

$$\lambda_8[\{t_{HL}d_H + T_{HL} - F_L\}P_{HL} - \{t_{LL}d_H + T_{LL} - F_L\}P_{LL}] = 0$$

$$\lambda_9[\{t_{LH}d_L + T_{LH} - F_H\}P_{LH} - \{t_{LL}d_L + T_{LL} - F_H\}P_{LL}] = 0$$

$$\lambda_{10}P_{HH} = 0$$

$$\lambda_{11}P_{HL} = 0$$

$$\lambda_{12}P_{LH} = 0$$

$$\lambda_{13}P_{LL} = 0$$

$$\lambda_{14}(1 - P_{HH}) = 0$$

$$\lambda_{15}(1 - P_{HL}) = 0$$

$$\lambda_{16}(1 - P_{LH}) = 0$$

$$\lambda_{17}(1 - P_{LL}) = 0$$

$$\lambda_{18}t_{HH} = 0$$

$$\lambda_{19}t_{HL} = 0$$

$$\lambda_{20}t_{LH} = 0$$

$$\lambda_{21} t_{LL} = 0$$

$$\lambda_{22}(n - t_{HH}) = 0$$

$$\lambda_{23}(n - t_{HL}) = 0$$

$$\lambda_{24}(n - t_{LH}) = 0$$

$$\lambda_{25}(n - t_{LL}) = 0$$

$$\lambda_n \geq 0 \ \forall n = 1, 2, ..., 25$$

First-Order-Conditions

$P_{ij}s$

$$\alpha_{HH}\{[n - t_{HH}]s - T_{HH}\} + (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)\{t_{HH}d_H + T_{HH} - F_H\} + \lambda_{10} - \lambda_{14} = 0$$

$$\alpha_{HL}\{[n - t_{HL}]s - T_{HL}\} + (\lambda_2 - \lambda_5 + \lambda_8)\{t_{HL}d_H + T_{HL} - F_L\} - \lambda_5 \Delta F + \lambda_{11} - \lambda_{15} = 0$$

$$\alpha_{LH}\{[n - t_{LH}]s - T_{LH}\} + (\lambda_3 - \lambda_6 + \lambda_9)\{t_{LH}d_L + T_{HL} - F_H\} - \lambda_6 \Delta c t_{LH} + \lambda_{12} - \lambda_{16} = 0$$

$$\alpha_{LL}\{[n - t_{LL}]s - T_{LL}\} + (\lambda_4 - \lambda_9 - \lambda_7 - \lambda_8)\{t_{LL}d_L + T_{LL} - F_L\} -$$

$$(\lambda_7 + \lambda_8)\Delta c t_{LL} - (\lambda_7 + \lambda_9)\Delta F + \lambda_{13} - \lambda_{17} = 0$$

$t_{ij}s$

$$-\alpha_{HH}P_{HH}s + (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)P_{HH}d_H + \lambda_{18} - \lambda_{22} = 0$$

$$-\alpha_{HL}P_{HL}s + (\lambda_2 - \lambda_5 + \lambda_8)P_{HL}d_H + \lambda_{19} - \lambda_{23} = 0$$

$$-\alpha_{LH}P_{LH}s + (\lambda_3 - \lambda_6 + \lambda_9)P_{LH}d_L - \lambda_6 \Delta c P_{LH} + \lambda_{20} - \lambda_{24} = 0$$

$$-\alpha_{LL}P_{LL}s + (\lambda_4 - \lambda_7 - \lambda_8 - \lambda_9)P_{LL}d_L - (\lambda_7 + \lambda_8)\Delta c P_{LL} + \lambda_{21} - \lambda_{25} = 0$$

$T_{ij}s$

$$-\alpha_{HH}P_{HH} + (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)P_{HH} = 0$$

$$-\alpha_{HL}P_{HL} + (\lambda_2 - \lambda_5 + \lambda_8)P_{HL} = 0$$

$$-\alpha_{LH}P_{LH} + (\lambda_3 - \lambda_6 + \lambda_9)P_{LH} = 0$$

$$-\alpha_{LL}P_{LL} + (\lambda_4 - \lambda_7 - \lambda_8 - \lambda_9)P_{LL} = 0$$

Knowing from the first order conditions for $T_{ij}s$, the first order conditions for t_{ij} can be expressed as:

$$\alpha_{HH}P_{HH}(d_H - s) + \lambda_{18} - \lambda_{22} = 0$$

$$\alpha_{HL}P_{HL}(d_H - s) + \lambda_{19} - \lambda_{23} = 0$$

$$\alpha_{LH}P_{LH}(d_L - s) - \lambda_6 \Delta c P_{LH} + \lambda_{20} - \lambda_{24} = 0$$

$$\alpha_{LL}P_{LL}(d_L - s) - (\lambda_7 + \lambda_8)\Delta c P_{LL} + \lambda_{21} - \lambda_{25} = 0$$

For all the f.o.cs for $t_{ij}s$:

$$\alpha_{HH}P_{HH}(d_H - s)t_{HH} = \lambda_{22}n$$

$$\alpha_{HL}P_{HL}(d_H - s)t_{HL} = \lambda_{23}n$$

$$\alpha_{LH}P_{LH}(d_L - s)t_{LH} - \lambda_6 \Delta c P_{LH} t_{LH} = \lambda_{24}n$$

$$\alpha_{LL}P_{LL}(d_L - s)t_{LL} - (\lambda_7 + \lambda_8)\Delta c P_{LL} t_{LL} = \lambda_{25}n$$

Times $P_{ij}s$ to f.o.c.s of $P_{ij}s$

$$P_{HH}\alpha_{HH}\{[n - t_{HH}]s - T_{HH}\} + (\lambda_1 + \lambda_5 + \lambda_6 + \lambda_7)P_{HH}\{t_{HH}d_H + T_{HH} - F_H\} - \lambda_{14} = 0$$

$$P_{HL}\alpha_{HL}\{[n-t_{HL}]s-T_{HL}\}+(\lambda_2-\lambda_5+\lambda_8)P_{HL}\{t_{HL}d_H+T_{HL}-F_L\}-\lambda_5 P_{HL}\Delta F-\lambda_{15}=0$$

$$\alpha_{LH}P_{LH}\{[n-t_{LH}]s-T_{LH}\}+(\lambda_3-\lambda_6+\lambda_9)\{t_{HL}d_L+T_{HL}-F_H\}-P_{LH}\lambda_6\Delta ct_{LH}-\lambda_{16}=0$$

$$\alpha_{LL}P_{LL}\{[n-t_{LL}]s-T_{LL}\}+(\lambda_4-\lambda_9-\lambda_7-\lambda_8)P_{LL}\{t_{LL}d_L+T_{LL}-F_L\}-$$

$$P_{LL}(\lambda_7+\lambda_8)\Delta ct_{LL}-(\lambda_7+\lambda_9)P_{LL}\Delta F-\lambda_{17}=0$$

FIRST ORDER CONDITIONS: for T_{ij},t_{ij},P_{ij} accordingly:

$$P_{HH}[(\lambda_1+\lambda_5+\lambda_6+\lambda_7)-\alpha_{HH}]=0$$

$$P_{HL}[(\lambda_2-\lambda_5+\lambda_8)-\alpha_{HL}]=0$$

$$P_{LH}[(\lambda_3-\lambda_6+\lambda_9)-\alpha_{LH}]=0$$

$$P_{LL}[(\lambda_4-\lambda_7-\lambda_8-\lambda_9)-\alpha_{LL}]=0$$

$$\alpha_{HH}P_{HH}(d_H-s)t_{HH}=\lambda_{22}n$$

$$\alpha_{HL}P_{HL}(d_H-s)t_{HL}=\lambda_{23}n$$

$$\alpha_{LH}P_{LH}(d_L-s)t_{LH}-\lambda_6\Delta cP_{LH}t_{LH}=\lambda_{24}n$$

$$\alpha_{LL}P_{LL}(d_L-s)t_{LL}-(\lambda_7+\lambda_8)\Delta cP_{LL}t_{LL}=\lambda_{25}n$$

$$P_{HH}\alpha_{HH}\{[ns-F_H+t_{HH}(d_H-s)\}-\lambda_{14}=0$$

$$P_{HL}\alpha_{HL}\{[ns-F_L+t_{HL}(d_H-s)\}-\lambda_5 P_{HL}\Delta F-\lambda_{15}=0$$

$$\alpha_{LH}P_{LH}\{[ns-F_H+t_{HL}(d_L-s)\}-P_{LH}\lambda_6\Delta ct_{LH}-\lambda_{16}=0$$

$$\alpha_{LL}P_{LL}\{[ns-F_L+t_{LL}(d_L-s)\}-$$

$$P_{LL}(\lambda_7+\lambda_8)\Delta ct_{LL}-(\lambda_7+\lambda_9)P_{LL}\Delta F-\lambda_{17}=0$$

Government's Justification For The Environment

By observing the f.o.c.s for T_{ij}s:

$$P_{HH}[(\lambda_1+\lambda_5+\lambda_6+\lambda_7)-\alpha_{HH}]=0$$

$$P_{HL}[(\lambda_2-\lambda_5+\lambda_8)-\alpha_{HL}]=0$$

$$P_{LH}[(\lambda_3-\lambda_6+\lambda_9)-\alpha_{LH}]=0$$

$$P_{LL}[(\lambda_4-\lambda_7-\lambda_8-\lambda_9)-\alpha_{LL}]=0$$

The optimal contract will be specified by the Kuhn-Tucker conditions as follows. By excluding the LL type from operation, the government could treat either HH, HL and LH as if they are all LH as long their are enjoying a positive utility or they could be all treated as HL.

Proof: The construction decision being random means: with probability $0<P_{ij}<1$, the government has type ij constructing the project.

From the complementary slackness conditions, this means there is one pair from $\lambda_{10}$and λ_{14}, $\lambda_{11}$and λ_{15}, $\lambda_{12}$and λ_{16}, $\lambda_{13}$and λ_{17} equals to zero at the same time.

If $\lambda_{10}=\lambda_{14}=0$

$$P_{HH}\alpha_{HH}(ns-F_H)+\lambda_{22}n=0$$

$$P_{HH}=\frac{\lambda_{22}n}{\alpha_{HH}(F_H-ns)}$$

and bring this back to the first order condition of t_{HH}, from above $\lambda_{22}>0$ by assumption and $t_{HH}=n$

$$\frac{\lambda_{22}n}{(F_H-ns)}(d_H-s)n=\lambda_{22}n$$

The equation stands if and only if $(d_H-s)n=F_H-ns$, which means $P_H=1$

And $0<P_{HH}<1$ by assumption. therefore a contradiction.

$$\alpha_{LL}P_{LL}\{[ns - F_L + t_{LL}(d_L - s)\} - P_{LL}(\lambda_7 + \lambda_8)\Delta c t_{LL} - (\lambda_7 + \lambda_9)P_{LL}\Delta F - \lambda_{17} = 0$$

If $0 < P_{LL} < 1$, then $\lambda_{17} = 0$

$$\alpha_{LL}P_{LL}[ns - F_L] + \lambda_{25}n = 0$$

And $0 < P_{HL} < 1$ by assumption, therefore a contradiction.

And the proof for P_{LH} and P_{HL} follows the same logic, therefore the mechanism is not random here but we will be able to add ex ante randomization into our mechanism.

Here follows the specific Kuhn-Tucker conditions.

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_5 = \lambda_6 = 0$$

$$\lambda_4 = 1$$

$$\lambda_7 = \alpha_{HH}$$

$$\lambda_8 = \alpha_{HL}$$

$$\lambda_9 = \alpha_{LH}$$

When $\alpha_{LL}(d_L - s) - (\alpha_{HH} + \alpha_{HL})\Delta c > 0$

$$\lambda_{10} = \lambda_{11} = \lambda_{12} = \lambda_{13} = 0$$

$$\lambda_{14} = \alpha_{HH}\{nd_H - F_H\}$$

$$\lambda_{15} = \alpha_{HL}\{nd_H - F_L\}$$

$$\lambda_{16} = \alpha_{LH}\{nd_L - F_H\}$$

$$\lambda_{17} = \alpha_{LL}\{nd_L - F_L)\} - (\alpha_{HH} + \alpha_{HL})n\Delta c - (\alpha_{HH} + \alpha_{LH})\Delta F$$

$$\lambda_{18} = \lambda_{19} = \lambda_{20} = \lambda_{21} = 0$$

$$\lambda_{22} = d_H - s$$

$$\lambda_{23} = d_H - s$$

$$\lambda_{24} = d_L - s$$

$$\lambda_{25} = \alpha_{LL}(d_L - s) - (\alpha_{HH} + \alpha_{HL})\Delta c$$

$$t_{HH} = t_{HL} = t_{LH} = t_{LL} = n$$

$$P_{HH} = P_{HL} = P_{LH} = P_{LL} = 1$$

A.6 The Abstract Setup and The Proof of Decomposibility of Chapter 2

In the previous section, we study how the uncertainty and the combinatorial structure of the optimization problems affect the solution to the optimization problem. Following the same spirit, we further generate the problem to talk more broadly about setting up mechanism to implement social choice functions.

First a few definitions:

A social choice function is a function f: $\Theta_1 \times ... \times \Theta_I \to X$ that, for each possible profile of the agents' types $(\theta_1, ..., \theta_I)$ assigns a collective choice $f(\theta_1, .., \theta_I) \in X$. Intuitively, a social choice function generates feasible allocations to every provided preference profile over individuals.

I first set up a communication mechanism to study the implementability of the social choice function when no uncertainty preserves. In the communication mechanism, the strategy profiles are consisted of messages $M(\theta)$ and the outcome is $g(\cdot)$. M-implementable is defined below. The important findings before (Green and Laffont 1986) is that the implementability of any collective decision rule is determined by the interaction of the allowable messages and the associated actions. If the messages are restricted in some circumstances, there is an enhanced potential to implement mutually beneficial course of action.

With the institutional setup, the mechanism is deliberatly splitting into two complementary submechanisms, without uncertainty, I claim there is no loss of generality to decompose one M-implementable mechanism into two complementary submechanisms to implement the same social choice function regardless of the information disclosure process.

Yet with the consideration of the uncertainty preserves in the mechanism, the implementability of the mechanism will indeed be affected by the information disclosure. Because with the uncertainty concern, the communication mechanism should employ a Bayesian setting. Hence the information disclosure process affect the Bayesian updating in the mechanism, hence the equilibrium strategies.

The strategy profile $s^*(\cdot) = (s_1^*(\cdot), ..., s_I^*(\cdot))$ is a Bayesian Nash equilibrium of mechanism $\Gamma = (S_1, ..., S_I, g(\cdot))$ if , for all i and all $\theta_i \in \Theta_i$,

$$E_{\theta_{-i}}[u_i(g(s_i^*(\theta_i), s_{-i}^*), \theta_i)|\theta_i] \geq E_{\theta_{-i}}[u_i(g(\hat{s}_i(\theta_i), s_{-i}^*), \theta_i)|\theta_i]$$

for all $\hat{s}_i \in S_i$.

The mechanism $\Gamma = (S_1, ..., S_I, g(\cdot))$ implements the social choice function $f(\cdot)$ in Bayesian Nash Equilbirum if there is a Bayesian Nash equilibrium of Γ, $s^*(\cdot) = (s_1^*(\cdot), ..., s_I^*(\cdot))$ such that $g(s^*(\theta)) = f()$ for all $\theta \in \Theta$.

The interesting thing about the Bayesian implementation is truth telling need only give agent i his highest payof averaging over all possible types θ_{-i} that might arise for the other agents. Hence how uncernty revealed (information disclosed) is closely related to the averaging process. Also notice the averaging process is specific to the implementation game behind instead of on the any social choice function.

The Model

Benchmark: A Single regulator With Full Report of All The Private Information

This is a regulator-agent problem. The agent's utility function depends on a pair of parameters or characteristics, $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ and a decision pair $(x_1, x_2) \in X$. Denote this utility $u(x_1, x_2, \theta_1, \theta_2)$. The agent observe (θ_1, θ_2). The regulator will announce (x_1, x_2) as a function of the report from the agent before (θ_1, θ_2) is observed.

First of all, several definitions.

Suppose the message correspondence is M: $\Theta_1 \times \Theta_2 \rightarrow \Theta_1 \times \Theta_2$, such that $\theta_1 \times \theta_2 \in M(\theta_1, \theta_2)$, $\forall(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$.

An outcome function is $g : \Theta_1 \times \Theta_2 \rightarrow X$, where X is the set of all possible outcomes.

Given the correspondence $M(\theta_1, \theta_2)$ the outcome function g induces a response rule $\Phi_g : \Theta_1 \times \Theta_2 \rightarrow \Theta_1 \times \Theta_2$, defined by:

$$\Phi_g(\theta_1, \theta_2) \in argmax_{m \in M(\theta_1, \theta_2)} u(g(m), \theta)$$

A social choice function f is defined as $f : \Theta_1 \times \Theta_2 \rightarrow X$ and f is $M(.,.)$ implementable iff $\exists$ an outcome function $g : \Theta_1 \times \Theta_2 \rightarrow X$ such that:

$$g(\Phi_g(\theta_1, \theta_2)) = f(\theta_1, \theta_2)$$

$\forall(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ and $\Phi_g(.,.)$ is an induced response rule.

A social choice function $f : \Theta_1 \times \Theta_2 \rightarrow X$ is truthfully $M(.,.)$-implementable iff there exists an outcome function $g^* : \Theta_1 \times \Theta_2 \rightarrow X$ such that, for any $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$,

$$g^*(\Phi_{g^*(\theta_1, \theta_2)}) = f(\theta_1, \theta_2)$$

and

$$\Phi_{g^*(\theta_1, \theta_2)} = (\theta_1, \theta_2)$$

Two regulators With Two different Private Information Report Requirements

This is a regulators-agent problem. The agent's utility function depends on a pair of parameters or characteristics, $(\theta_1, \theta_2) \in \Theta_1 \times \Theta_2$ and a decision pair $(x_1, x_2) \in X$. Denote this utility $u(x_1, x_2, \theta_1, \theta_2)$. The agent observe (θ_1, θ_2). The regulator A will announce x_1 as a function of the report from the agent before (θ_1, θ_2) is observed, then the regulator

B will announce x_2 as a function of the report from the agent before $(\theta_1, \theta_2)$is realized. Regulator A and regulator B do not communicate. regulator A first makes the decision, then regulator B.

There are two different report requirements: first only report θ_1 to regulator A and report θ_2 to regulator B; second report (θ_1, θ_2) to both regulators.

I will mimic the benchmark model to formulate a model with two regulators, and each of them is responsible for only part of the corrdination plan.

The outcome X is a product of $X = X_1 \times X_2$, where $x_1 \in X_1$ and $x_2 \in X_2$.

The characteristics $\Theta_1 \times \Theta_2$ is a product, where $\theta_1 \in \Theta_1$ and $\theta_2 \in \Theta_2$.

The following conditions say given only partial information Θ_i, a social choice function f_i is sufficient to induce all outcomes on X_i, where $i = 1, 2$

A partial mechanism for regulator A is $(M(\theta_1, .), g_1)$ consists of a correspondence $M(\theta_1, .) : \Theta_1 \to \Theta_1$ such that $\theta_1 \in M(\theta_1, .)$ for all $\theta_1 \in \Theta_1$ and an outcome function $g_1 : \Theta_1 \to X_1$.

An outcome function is $g_1 : \Theta_1 {\to} X_1$, where X_1 is one part of all possible outcomes.

Given the correspondence $M(\theta_1, .)$ the outcome function g_1 induces a response rule $\Phi_{g_1} : \Theta_1 {\to} \Theta_1$, defined by:

$$\Phi_{g_1}(\theta_1, .) \in argmax_{m_1 \in M(\theta_1, .)} u(g_1(m_1), \theta)$$

A partial social choice function f_1 for regulator A is defined as $f_1 : \Theta_1 {\to} X_1$ and f_1 is $M(\theta_1, .)$ implemenatatable iff $\exists$ an outcome function $g_1 : \Theta_1 {\to} X_1$ such that:

$$g_1(\Phi_{g_1}(\theta_1, .)) = f_1(\theta_1)$$

$\forall \theta_1 \in \Theta_1$ and $\Phi_{g_1}(\theta_1, .)$ is an induced response rule.

Similarly the other complementary regulator B's social choice function will have a similar setup.

A partial mechanism for regulator B is $(M(., \theta_2), g_1)$ consists of a correspondence

$M(.,\theta_2) : \Theta_2 \rightarrow \Theta_2$ such that $\theta_2 \in M(.,\theta_2)$ for all $\theta_2 \in \Theta_2$ and an outcome function $g_2 : \Theta_2 \rightarrow X_2$.

An outcome function is $g_2 : \Theta_2 \rightarrow X_2$, where X_2 is the other complementary part of all possible outcomes.

Given the correspondence $M(.,\theta_2)$ the outcome function g_2 induces a response rule $\Phi_{g_2} : \Theta_2 \rightarrow \Theta_2$, defined by:

$$\Phi_{g_2}(.,\theta_2) \in argmax_{m_2 \in M(.,\theta_2)} u(g_2(m_2),\theta)$$

A partial social choice function f_2 for regulator B is defined as $f_2 : \Theta_2 \rightarrow X_2$ and f_2 is $M(.,\theta_2)$ implemenatable iff $\exists$ an outcome function $g_2 : \Theta_2 \rightarrow X_2$ such that:

$$g_2(\Phi_{g_2}(.,\theta_2)) = f_2(\theta_2)$$

$\forall (.,\theta_2) \in \Theta_2$ and $\Phi_{g_2}(.,\theta_2)$ is an induced response rule.

So far, we can conclude that:

$$\Phi_{g_1}(\theta_1,.) \times \Phi_{g_2}(.,\theta_2) = \Phi_{g_1}(\theta_1,\theta_2) \times \Phi_{g_2}(\theta_1,\theta_2)$$

A social choice function $f_1 : \Theta_1 \rightarrow X_2$ is truthfully $M(\theta_1,.)$-implementable iff there exists an outcome function $g_1^* : \Theta_1 \rightarrow X_1$ such that, for any $(\theta_1,.)$in Θ_1,

$$g_1^*(\Phi_{g_1^*(\theta_1,.)}) = f_1(\theta_1,.)$$

and

$$\Phi_{g_1^*}(\theta_1,.) = \theta_1,$$

A social choice function $f_2 : \Theta_2 \rightarrow X_2$ is truthfully $M(.,\theta_2)$-implementable iff there exists an outcome function $g^* : \Theta_1 \times \Theta_2 \to X_2$ such that, for any (θ_1, θ_2)in $\Theta_1 \times \Theta_2$,

$$g_2^*(\Phi_{g_2^*(.,\theta_2)}) = f_2(., \theta_2)$$

and

$$\Phi_{g_2^*}(., \theta_2) = \theta_2$$

Decomposible Communication Mechanisms with Full Private Information Disclosure

I claim: With a utility function that is convex and compact in the outcomes X, a M-truthfully implementable social function is $M_1 \times M_2$- truthfully implementable.

Information disclosure should be complete to the dual mechanism designers' program.

The decomposibility of M-truthfully implementability to complementary submechanisms $M_1 \times M_2$ is guranteed by compactness and convexity of the outcome space.

Proof Task One: Sufficiency

With restrictions on the preference, a social choice function is truthfully implementable through a single mechanism designer implies the social choice function is truthfully implementable through full report to dual mechanism designers.

Proof Task Two: Necessity

With restrictions on preferences, a social choice function is truthfully implementable through full report to dual mechanism designers implies the social choice function is truthfully implementable through a single mechanism designers.

A.6.1 Model Priliminaries

Environment: one agent, dual cooperative mechanism designers v.s. single designer.

Presumptions: restrictions on the agent's utility function– compact and convex in outcomes.

Information disclosure: full report.

Institution environment: cooperative dual mechanism designers v.s centralized single designer

A.6.2 Definitions

For dual mechanism designers under full report: $M^2(\theta_1, \theta_2)$

- Define outcomes for each mechanism designer, and the outcome under the mechanism.

Outcomes are: $X = X_1 \times X_2$

Outcome function for each mechanism designer

$$g_1 : \Theta_1 \times \Theta_2 \to X_1$$

$$g_2 : \Theta_1 \times \Theta_2 \to X_2$$

The outcome funtion for the mechanism:

$$g_1 \times g_2 : [\Theta_1 \times \Theta_2]^2 \to X_1 \times X_2$$

- Define the social choice functions

A social choice function of the mechanism $f : \Theta_1 \times \Theta_2 \to X_1 \times X_2$ is separation report implentable iff there exists a correspondence of outcome functions:

$$g_1 \times g_2 : [\Theta_1 \times \Theta_2]^2 \to X_1 \times X_2$$

such that,

$$g_1(\phi_{g_1}(\theta_1, \theta_2)) = f_1(\theta_1, \theta_2)$$

$$g_2(\phi_{g_2}(\theta_1, \theta_2)) = f_2(\theta_1, \theta_2)$$

for any (θ_1, θ_2) in $\Theta_1 \times \Theta_2$ where $\phi_{g_i}(\theta_1, \theta_2)$ is the induced response rule:

$$\phi_{g_i}(\theta_1, \theta_2) \in argmax_{(\hat{\theta_1}, \hat{\theta_2}) \in M(\theta_1, \theta_2)} u(g_i(\hat{\theta_1} \times \hat{\theta_2}) \times g_{-i}(\hat{\theta_1} \times \hat{\theta_2}), \theta_1 \times \theta_2)$$

$$\forall i; -i \in \{1, 2\}$$

And also:

$$f_1(\theta_1, \theta_2) \times f_2(\theta_1, \theta_2) = X$$

Proposition 14. *Necessity*

A social choice function f is truthfully $M(\theta_1, \theta_2) \times M(\theta_1, \theta_2)$- implementable to dual mechanism designers implies the social choice function f is truthfully $M(\theta_1, \theta_2)$-implementable to the single mechanism designer.

Proof. If a social choice function $f : \Theta_1 \times \Theta_2 \to X_1 \times X_2$ is truthfully $M(\theta_1, \theta_2) \times M(\theta_1, \theta_2)$-implementable: there exists an outcome correspondence

$$g_1 \times g_2 : [\Theta_1 \times \Theta_2]^2 \to X_1 \times X_2$$

such that, for any (θ_1, θ_2) in $\Theta_1 \times \Theta_2$,

$$g_{1*}(\phi_{g_{1*}(\theta_1, \theta_2)}) = f_1(\theta_1, \theta_2)$$

$$g_{2*}(\phi_{g_{2*}(\theta_1, \theta_2)}) = f_2(\theta_1, \theta_2)$$

$$\phi_{g_{1*}(\theta_1, \theta_2)} = (\theta_1, \theta_2)$$

$$\phi_{g_{2*}(\theta_1, \theta_2)} = (\theta_1, \theta_2)$$

$$\phi_{g_1}(\theta_1, \theta_2) \in argmax_{(\hat{\theta_1} \times \hat{\theta_2}) \in M(\theta_1 \times \theta_2)} u(g_1(\hat{\theta_1} \times \hat{\theta_2}) \times g_2(\hat{\theta_1} \times \hat{\theta_2}), \theta_1 \times \theta_2)$$

Given the opitmal of the response function, enriching the messages sent to deparment does not change the outcome. The same is true for the messages sent to department 2.

Since enriching the messsages sent would not affect the outcome, without the loss of generality, the outcome will remain optimal no matter how many times the truth were reported.

Such social choice function f is truthfully $M(\theta_1, \theta_2)$-implementable to single mechanism designers: there exists an outcome correspondence $g_* : [\Theta_1 \times \Theta_2] \to X_1 \times X_2$ such that, for any (θ_1, θ_2) in $\Theta_1 \times \Theta_2$,

$$g^*(\phi_{g^*(\theta_1,\theta_2)}) = f(\theta_1, \theta_2)$$

$$\phi_{g^*(\theta_1,\theta_2)} = (\theta_1, \theta_2)$$

$$\forall(\theta_1, \theta_2)$$

$\square$

Lemma 14. *Every $g_{1*} \times g_{2*}$ well-defined in the separation reporting schemes in $M_1(\theta_1, \theta_2) \times M_2(\theta_1, \theta_2)$ setting and have a TRUTHFUL counter part in $M(\theta_1, \theta_2)$.*

$$f_1(\theta_1, \theta_2) \times f_2(\theta_1, \theta_2) = X_1 \times X_2 = X$$

Recall, the outcome correspondence $g_{1*} \times g_{2*} : [\Theta_1 \times \Theta_2]^2 \to X_1 \times X_2$ such that, for any (θ_1, θ_2) in $\Theta_1 \times \Theta_2$,

$$g_{i*}(\phi_{g_{1*}(\theta_1,\theta_2)}) = f_i(\theta_1, \theta_2)$$

$$\phi_{g_{i*}(\theta_1,\theta_2)} = (\theta_1, \theta_2)$$

$$\forall i \in \{1, 2\}$$

In other words. the equiblium outcome $g_{1*} \times g_{2*}$ in the complementary submechanisms $\Gamma_1 \times \Gamma_2$ truthfully implement the social choice function f.

Since $u(x_1, x_2, \theta_1, \theta_2)$ is convex and compact in the their messages report $_1 \times x_2$.

$$ru((x_1,x_2),\theta_1,\theta_2)+(1-r)u((x_1,x_2),\theta_1,\theta_2)\leq u(r(x_1,x_2)+(1-r)(x_1,x_2),\theta_1,\theta_2)$$

$$\leq u(rx_1+(1-r)x_1,rx_2+(1-r)x_2,\theta_1,\theta_2)$$

The first inequality holds because the utility function is convex in (x_1,x_2) and the second inequality holds because the utility function is compact in the outcome correspondence, and denote the convex hull Co, therefore $Co(X_1)\times Co(X_2)\subset Co(X_1\times X_2)$

Every $g_{1*}\times g_{2*}$ in $M_1(\theta_1,\theta_2)\times M_2(\theta_1,\theta_2)$ setting have a TRUTHFUL counter part in $M(\theta_1,\theta_2)$ is proved.

Proposition 15. *Sufficiency*

If $u(x_1,x_2,\theta_1,\theta_2)$, is compact and convex in both x_1 and x_2, a social choice function f is truthfully $M(\theta_1,\theta_2)$- implementable to dual mechanism designers implies the social choice function f is truthfully $M_1(\theta_1,\theta_2)\times M_2(\theta_1,\theta_2)$-implementable to dual mechanism designers.

Proof. A social choice function $f:\Theta_1\times\Theta_2\to X$ is truthfully M-implementable iff there exists an outcome function $g^*:\Theta_1\times\Theta_2\to X$ such that, for any $(\theta_1,\theta_2)\in\Theta_1\times\Theta_2$,

$$g^*(\Phi_{g^*(\theta_1,\theta_2)})=f(\theta_1,\theta_2)=x_1^*\times x_2^*$$

and

$$\Phi_{g^*(\theta_1,\theta_2)}=(\theta_1,\theta_2)$$

Where $x_1^*\times x_2^*$ is the equilibrium outcome correspondence.

If a social choice function $f:\Theta_1\times\Theta_2\to X_1\times X_2$ is truthfully $M_1(\theta_1,\theta_2)\times M_2(\theta_1,\theta_2)$-implementable: there exists an outcome correspondence

$$g_1\times g_2:[\Theta_1\times\Theta_2]^2\to X_1\times X_2$$

such that, for any (θ_1, θ_2) in $\Theta_1 \times \Theta_2$,

$$g_{1*}(\phi_{g_{1*}(\theta_1,\theta_2)}) = f_1(\theta_1, \theta_2)$$

$$g_{2*}(\phi_{g_{2*}(\theta_1,\theta_2)}) = f_2(\theta_1, \theta_2)$$

$$\phi_{g_{1*}(\theta_1,\theta_2)} = (\theta_1, \theta_2)$$

$$\phi_{g_{2*}(\theta_1,\theta_2)} = (\theta_1, \theta_2)$$

$$u(g_{1*}(\theta_1 \times \theta_2) \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2) \geq u(g_1(\hat{\theta_1} \times \hat{\theta}_2) \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2)$$

$$\forall g_1(\hat{\theta_1})$$

The inequality stands by the definition of separation truthful implementability.

Then such inequality holds when $g_1(\hat{\theta_1}) = x_1^*$

$$u(g_{1*}(\theta_1 \times \theta_2) \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2) \geq u(x_1^* \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2)$$

Also by the optimality of x_1^* of M-implementability, the following inequality holds:

$$u(g_{1*}(\theta_1 \times \theta_2) \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2) \leq u(x_1^* \times g_2(\hat{\theta_1} \times \hat{\theta}_2), \theta_1; \theta_2)$$

Hence

$$g_{1*}(\theta_1) = x_1^*$$

Apply the same logic,

$$g_{2*}(\theta_1) = x_2^*$$

By lemma 1 the correspondence $g_1(\hat{\theta}_1 \times \hat{\theta}_2) \times g_2(\hat{\theta}_1 \times \hat{\theta}_2)$ is well defined given (x_1, x_2)

Hence truthfully reporting the complementary truth to different departments according truthfully implement the social function.

$\square$

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